FUZZY LOGIC CONTROL OF A CIRCULAR CYLINDER VORTEX SHEDDING MODEL

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A short computational program was undertaken to evaluate the effectiveness of a closed-loop control strategy for the stabilization of an unstable bluff-body flow. In this effort, the nonlinear one-dimensional Ginzburg-Landau wake model at 20% above the analogue of critical wake Reynolds number was studied. The numerical model, which is a nonlinear partial differential equation with complex coefficients, was solved using the FEMLAB®/MATLAB® package and validated by comparison with published literature. The closed loop system was controlled using a conventional proportional-integral-derivative (PID) controller as well as a nonlinear fuzzy controller. A single sensor is used for feedback, and the actuator is represented by altering the boundary conditions of the cylinder. The closed-loop feedback strategy examined included proportional control, PID control and fuzzy logic control. Results show that the fuzzy logic control provided the best response with respect to the onset of vortex suppression, the settling times, as well as the control effort applied. The results indicate that for a single sensor scheme, the increase in the sophistication of the control results in significantly shorter settling times. However, there is only a marginal improvement concerning the suppression of the wake at higher Reynolds numbers.

Nomenclature

$A(x,t)$	Complex amplitude of the Ginzburg-Landau model
C_{ij}	Coefficients of the linear stochastic estimator
c_n, c_d	Complex coefficients of the Ginzburg-Landau model
$F(x,t)$	External forcing in the Ginzburg-Landau model
$G_s(t)$	Time-Varying gains
K_P, K_I, K_D	Proportional, Integral and Derivative Gain of the PID controller
$N(t)$	Noise parameter in the Ginzburg-Landau model
U	Advection speed
x_s	Single sensor location used for feedback
x	Spatial coordinate
$\alpha_F(t)$	Gain varying parameter that is the output of the fuzzy algorithm
δ	Dirac delta function
$\mu(x)$	Wake growth rate parameter
μ_{crit}	Value above which the self-excited oscillations begin
μ'	Slope of the wake growth rate parameter with respect to x
μ_0	Analogue of wake Reynolds number. Also referred to in text as "Reynolds number"

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Introduction

Although there has been intense research in aerodynamic flow control, full implementation of the active manipulation of a flow field has remained elusive. However, with the surge of new technologies in the areas of sensors, actuators, real-time data processing, and nonlinear feedback control, the dawn of the “closed-loop era” is breaking. The necessary pieces seem to exist, but assembling them together *effectively* is the current challenge. This area of research is multi-disciplinary in nature, merging the fields of fluid mechanics, controls, simulations, data processing, structures, sensing and actuation.

The phenomenon of vortex shedding behind bluff bodies has been a subject of extensive research. Many flows of engineering interest produce the phenomenon of vortex shedding and the associated chaotic response. Applications include aircraft and missile aerodynamics, marine structures, underwater acoustics, and civil and wind engineering. The ability to control the wake of a bluff body could be used to reduce drag, increase mixing and heat transfer, and enhance combustion.

Flows with absolute instabilities behind bluff bodies, an archetype of which is the cylinder wake, demonstrate self-excited oscillations even when all sources of noise are removed[1]. Above a critical Reynolds number (Re ~ 47), non-dimensionalized with respect to freestream speed and cylinder diameter, in the wake of a two-dimensional (2D) cylinder, a significant region of local absolute instability occurs which results in a global flow instability, also known as the Karman vortex street. Figure 1, a picture taken at the center line of an unforced cylinder wake at the US Air Force Academy’s water tunnel, shows the Karman vortex street at Re = 120.

The complex Ginzburg-Landau (GL) equation, with suitable coefficients, models well the absolute instability of bluff-body wakes. The one dimensional GL equations provide useful insight for the description of global modes for purely 2D shedding where the spatial coordinate in the GL equation coincides with the streamwise direction[2].

The 1D GL equation, which is derivable from the Navier-Stokes equations, can be modeled to contain all of the stability features of the 2D cylinder wake pertinent to control. Furthermore, the GL model is frequently used in the literature for wake control studies and has been shown to allow semi-qualitative predictions of the wake with feedback[3,4]. An attractive characteristic of the GL model is that it is relatively straightforward to integrate numerically and allows *relatively rapid* prototyping of control strategies.

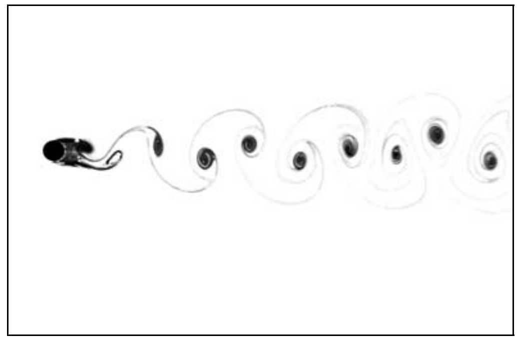


Fig. 1: Unforced Flow along the Center Line of a Circular Cylinder (Re = 120, Diameter D = 4.97 mm)

In this effort, three closed-loop control laws for the Ginzburg-Landau equation are developed: (a) Simple Proportional Gain based on the approach adopted by Park, Ladd and Hendricks[4] and Gillies[3], (b) Proportional-Integral-Derivative (PID) control and (c) Fuzzy Logic variable gain techniques. The main objective of the controller is to extend the value of the analogue of critical wake Reynolds number as much as possible utilizing a single sensor. The analogue of critical wake Reynolds number, referred to in this paper as the Reynolds number, is the value at which unsteady vortex shedding begins.

The paper is organized as follows: The next section describes the research objective and uniqueness of the developed approach. The Ginzburg-Landau equation is presented in the following section, and the FEMLAB® model is described subsequently. Then, the fuzzy logic controller is developed, followed by a comparison of the results for the developed controllers. The conclusions to date of this research effort are summarized in the final section.

Research Objective

The main objective of this research effort is to develop a robust strategy to suppress the Karman vortex street, as modeled using the Ginzburg-Landau equations utilizing feedback control. There has been some research on closed-loop control of the Ginzburg-Landau model using a simple proportional fixed gain approach by Park, Ladd and Hendricks[4], Roussopoulos and Monkewitz[2] and Gillies[3]. The current research effort is unique in that a variable gain strategy based on the inherently robust fuzzy logic control methodology is introduced for the first time. An attempt will be made to examine the effectiveness of a variable versus fixed gain strategy on closed-loop behavior.

The Ginzburg-Landau Wake Model

The 1D GL equation chosen is based on Gillies [3,5] as follows:

$$\frac{\partial A}{\partial t} + U\frac{\partial A}{\partial x} = \mu(x)A + (1 + jc_d)\frac{\partial^2 A}{\partial x^2} - (1 + jc_n)|A|^2 A + F(x,t) \tag{1}$$

where A(x,t) is the complex amplitude and U, c_d, c_n and $\mu(x)$ are real. F(x,t) incorporates the effects of feedback and noise. The stability of the GL 'wake' is defined by the growth parameter

$$\mu(x) = \mu_o + \mu' \cdot x \tag{2}$$

where μ_o is similar to a Reynolds number based on the cylinder diameter and μ' is the slope of the wake growth rate parameter with respect to x. For $\mu' < 0$ the stability features of this 'prototype' wake are similar to the stability features of a 2D cylinder wake, i.e. the emergence of a self-excited unstable response followed by a limit cycle. This non-linear instability is presented in Fig. 2 for 5% above the critical value of μ_o and x = 9.2. The parameter μ_o reaches its critical value at the Hopf bifurcation, when the limit cycle first appears.

For the flow around cylinders, several forcing techniques affect the behavior of the flow; however, the wake response to forcing is similar for each, whether vibration of the cylinder in the direction parallel to or perpendicular to the mean flow, rotation of the cylinder or alternate blowing and suction at the separation points[3] . Control forcing of the wake will be introduced into the GL equation by an actuation function placed in the near wake, namely F(x,t), using simple delta functions. The actuator will provide a step perturbation to the complex amplitude over the spatial actuation range: $0 < x_a < 2.0$.

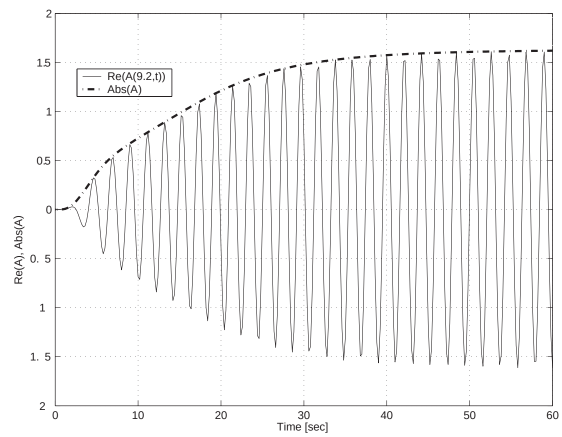


Figure 2: Uncontrolled Wake Signal at 5% above the critical value

The effects of feedback and noise may be incorporated into the GL equation in two ways. The first method, suggested by Roussopoulos and Monkewitz[2] and Gillies[3], involves modeling of the actuator as a delta function forcing at a fixed spatial location, x_a, and a sensor location at x_s, as shown in Equation (3):

$$F(x,t) = \sum_{s=1}^{n}[G_s(t)A(x_s,t)].[\delta(x - x_a)] + N(t)\delta(x - x_a) \tag{3}$$

The signals from n sensors are fed back with time-varying gains $G_s(t)$. The second term, N(t), describes the noise added to the system. Noise may be modeled with a random number generator of adjustable amplitude.

An alternative method, utilized in this study, was proposed by Park, Ladd and Hendricks[4]. This approach involves the modeling of an active boundary condition at x = 0 for the Ginzburg-Landau equation as follows:

$$A(0, t) = \alpha(t)A(x_s,t) \tag{4}$$

where $\alpha(t)$ may be time-invariant as in the case of Park, Ladd and Hendricks[4] and Gillies[3] or it may be a variable gain methodology. In this effort, both fixed and variable gain strategies are investigated. All other simulation parameters that define the Ginzburg-Landau model are based on Park, Ladd and Hendricks[4] and Gillies'[3] model to enable comparison with literature:

- 1D domain $0 < x < 120$
- Boundary conditions: $A(0,t) = 0$ (which simulates the cylinder body); $A(120,t) = 0$
- Fixed Parameters: $U = 5$; $\boldsymbol{\mu'} = -0.0434$; $c_d = 1$; $c_n = 0$.

Computational Model

After writing the GL equation, which is a non-linear partial differential equation with complex coefficients, the next step is to solve it numerically. After a survey of the market for an appropriate solver, FEMLAB®[6] was selected. FEMLAB® is an interactive environment for finite-element modeling and simulating scientific and engineering problems based on partial differential equations (PDEs). FEMLAB's® ability to arbitrarily define and couple any number of nonlinear PDEs, as well as work within the MATLAB®/SIMULINK® environment, makes it an attractive tool for studying fluid-control interaction. Furthermore, the solution of the Ginzburg-Landau equation is provided by FEMLAB® as a benchmark in their model library[6].

The details of the FEMLAB® model of the Ginzburg-Landau equation are as follows:

- **Element Type:** 'solid1(x)' – creates a 1-D solid object that spans all the coordinate values in the vector x (1-D domain $0 < x < 120$)
- **Number of Elements:** 300
- **Number of Nodes:** 301
- **Boundary Conditions:** $A(0,t) = 0$; $A(120,t) = 0$
- **Time-Step:** 0.2 units
- **Total Run-time:** 60 units
- **Initial condition:** $A(x,0) = 0.0001$

Gillies[3] reported that for his model the wake exhibits self-excited wake oscillations above $\mu_o = \mu_{crit} = 3.43$. The above coefficients for the study of the spatially developing flows, based on the Ginzburg-Landau model, were first introduced by Park, Ladd and Hendricks[4]. The value of μ_{crit} is of importance; therefore one of the aims of the current work was to arrive at the same value of μ_{crit} using the model currently developed (using FEMLAB®) as Gillies[3]. Simulation results show that the FEMLAB® model predicts the value of $\mu_{crit} = 3.43$ to within 0.3% of that obtained by Gillies[3] based on the same coefficients of the Ginzburg-Landau equation. Figure 2 shows the temporal plot of the real part and the absolute modulus of $A(x, t)$ at 5% above the critical value of μ_o. Figure 3 displays the spatial plots of the real part and the absolute modulus of $A(x, t)$ at steady state at 5% above the critical value of μ_o. Since the parameter μ_o is above the critical value, the amplitude initially grows exponentially, and then it almost equilibrates at a saturated level, or limit cycle, due to the stabilizing cubic nonlinearity. Furthermore, the FEMLAB® results shown in Fig. 2 and Fig. 3 compare very well with those presented by Park, Ladd and Hendricks[4].

Following the successful modeling of the open-loop behavior of the Ginzburg-Landau equation using FEMLAB®, the model was exported to SIMULINK® for closed loop studies. In addition, the SIMULINK® model provides for the "placement" of several sensors in the wake ($0 < x < 120$). Currently, the output of a single sensor, strategically positioned as proposed by Gillies[3] at $x = 3.2$ (Node 9) is incorporated for the feedback control. A sensitivity study was conducted with the single sensor, used for feedback, placed at nodes 7 ($x = 2.4$) to 12 ($x = 4.4$). Node 9 was found to be most compatible with the findings of Gillies[3] concerning single sensor placement. An additional sensor, introduced at $x = 10.0$, serves as an observer to ensure that the suppression of the vortex shedding is global for the entire range of x and not local in nature in the vicinity of the sensor used for feedback.

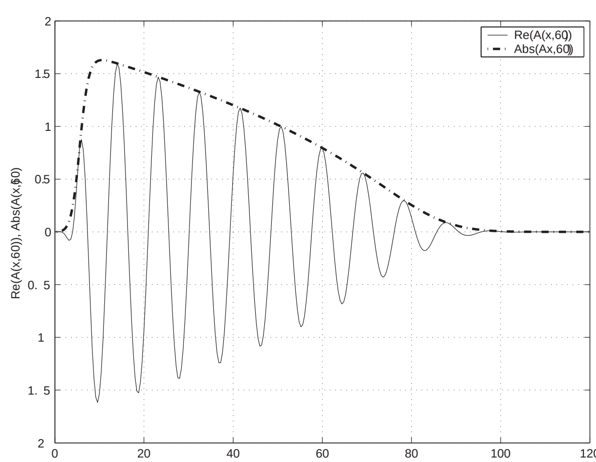


Figure 3: Uncontrolled Wake Signal at 5% above Critical

Fuzzy Logic Control

The uncertainties resulting from modeling errors inherent with wake flow dynamics and the effects of various disturbances make *robustness* an essential attribute of the control system. In order to circumvent many of the modeling and control problems mentioned, an estimator/controller strategy based on inherently robust soft computing techniques such as fuzzy logic has been selected. The main advantages of using a fuzzy approach are the relative ease and simplicity of implementation and its robustness. The gains of the fuzzy controller may be adapted to provide fairly fast control for large deviations of the measured state of the plant from the desired state and a minor amount of control for small deviations.

The successful implementation of a fuzzy logic controller depends, among other design aspects, on the heuristic rule base from which control actions are derived. In order to obtain the required heuristic physics-based insight, a single degree of freedom system based on optimal control theory was analytically examined to observe the characteristics of a minimum time solution. Based on this analysis, Cohen, Weller and Ben-Asher[7] introduced a fuzzy logic non-linear mapping function which has the potential of being a universal approximator to emulate the above minimum time solution. The resulting rule base, detailed in Cohen, Weller and Ben-Asher[7], is the core of the control law that is applied to the control of the Ginzburg-Landau equation presented herein.

Cohen, Weller and Ben-Asher[7] proposed an effective means of controlling a second-order systems by introducing a variable damping strategy which is implemented in the form of a fuzzy logic algorithm. This approach has been successfully applied to vibration suppression of flexible structures[8] and for active suppression of aircraft cabin noise that is induced by structure borne vibration[9]. Furthermore, the above method has been demonstrated experimentally on smart structures at the Technion, Israel Institute of Technology[10].

The fuzzy controller is of the form:

$$F_{Fuzzy} = \alpha_F(t).F_{PID} \tag{5}$$

where

$$F_{PID} = -K_p\, A(x_s,t) - K_I \int_0^t A(x_s,t)dt - K_D \frac{d(A(x_s,t))}{dt} \tag{6}$$

is the conventional PID controller and $\alpha_F(t)$ is the gain varying parameter that is the output of the fuzzy algorithm. The gains of the above PID controller (K_P, K_I and K_D) may be varied in real-time to provide fairly fast control for large deviations of the measured state of the plant from the desired state and a minor amount of control for small deviations. This adaptation strategy is implemented using fuzzy logic control and is based on successful implementation of a class of linear second-order systems[7]. The fuzzy controller is implemented as a 25-rule Mamdani fuzzy system with 2 inputs and 1 output as follows:

The two inputs into the fuzzy algorithm:

$$A(x_s,t), \frac{d(A(x_s,t))}{dt}$$

Single output from the fuzzy algorithm: $\alpha_F(t)$

As Table 1 shows, five membership functions are used to describe each of the input and output parameters. The respective membership functions for the inputs / output parameters are obtained after a tuning process. The fuzzy adaptation strategy is based on rules of the form "if...then..." that convert inputs to a single output, i.e. conversion of one fuzzy set into another. Heuristic rules based on previous experience are coupled with fuzzy reasoning whereby *large* values of the inputs require a *lightly* damped system, which would provide quick rise times. However, when the plant state is in the vicinity of the desired state, the damping factor is *large* to reduce the overshoot and steady state error. The Rule-Base is comprised of a set of 25 rules. A typical rule may be read as: If $A(x_s,t)$ is **Zero** *and* $d(A(x_s,t))/dt$ is **Zero**, then $\alpha_F(t)$ is EXTRA LARGE.

Simulation Results

The results obtained using a fuzzy control algorithm, were compared to those obtained using a PID controller. The PID controller is the special case when $\alpha_F(t)=1$. The values of the gains of the PID controller are: $K_P = 0.09$, $K_I = 0.0$, and $K_D = 0.06$. In order to examine the effectiveness of the variable gain strategy based on fuzzy logic control, results are compared to those obtained from the fixed gain controllers in the literature as well as the developed PID controller.

	$A(x_s,t)$ **Negative**	$A(x_s,t)$ **Negative Small**	$A(x_s,t)$ **Zero**	$A(x_s,t)$ **Positive Small**	$A(x_s,t)$ **Positive**
$d(A(x_s,t))/dt$ **Positive**	**VERY SMALL**	**SMALL**	**MEDIUM**	**SMALL**	**VERY SMALL**
$d(A(x_s,t))/dt$ **Positive Small**	**VERY SMALL**	**MEDIUM**	**LARGE**	**MEDIUM**	**VERY SMALL**
$d(A(x_s,t))/dt$ **Zero**	**MEDIUM**	**LARGE**	**EXTRA LARGE**	**LARGE**	**MEDIUM**
$d(A(x_s,t))/dt$ **Negative Small**	**VERY SMALL**	**MEDIUM**	**LARGE**	**MEDIUM**	**VERY SMALL**
$d(A(x_s,t))/dt$ **Negative**	**VERY SMALL**	**SMALL**	**MEDIUM**	**SMALL**	**VERY SMALL**

Table 1: Rule Base for the Fuzzy Logic Control Law for $\alpha_F(t)$

	Stabilization above $\mu_{crit}(P)$ [%]	**Control Effort at $\mu_o = 3.567$**	**Settling Time at $\mu_o = 3.567$ [sec]**
PID Control	100	0.0040	600
Fuzzy Logic Control	125	0.0029	200

Table 2: Comparison of results for PID controller and Fuzzy Controller

A linearization of the 300-node FEMLAB® model was exported to SIMULINK® for the closed-loop studies. The following parameters are defined:

- Settling Time – Time taken until:

  $abs[A(x_s,t)] \leq 0.05\ max[A(x_s,t)]$

- Sensor Reading – $A(x_s,t)$ at $x_s = 3.2$ (Node 9)
- Parameter $\mu_{crit}(P)$ – Maximum value of μ_o that still provides stable closed-loop system for proportional control - $\mu_o = 3.50$

The time histories of Re(A(10,t)) and the respective control input for the PID controller at $\mu_o = 3.578$ are presented in Fig. 4. It was interesting to note that the control input has stabilized based on a sensor reading at $x_s = 3.2$. However, the sensor signal placed at x = 10.0 indicates that the wake has not stabilized. The linearized wake was diverging and that suggests that a mere shifting around of the flow pattern has occurred without suppressing the wake along the x-dimension as desired. For the same value of $\mu_o = 3.578$, the entire wake is stabilized using the fuzzy logic controller (see Fig. 5). Finally, the fuzzy logic controller goes unstable at $\mu_o = 3.585$. Table two compares the results obtained using a fuzzy controller to those obtained using PID control. These results may be summarized as follows:

PID vs. Proportional Control - The PID control delays the onset of vortex shedding at twice the improvement obtained using P control. The settling time compared to literature (Park, Ladd and Hendricks[4] using just Proportional feedback), shows an improvement of an order of magnitude.

Fuzzy vs. PID - The onset of vortex shedding is further delayed by another 25%. The settling time for the stable condition near "Re Critical" for the fuzzy controller is a third in comparison. The overall control effort (integral of control input over time for the same maximum allowable actuation) used by the PID controller is 38% more than the fuzzy design.

Discussion and Conclusions

The current effort concerns the vortex suppression of the cylinder wake modeled using the Ginzburg-Landau equations. The problem is high-dimensional in character with the characteristic Hopf bifurcation that represents the vortex shedding. A SIMULINK® model was developed using FEMLAB® based on finite element analysis. The closed loop system was controlled using a conventional PID controller as well as a nonlinear fuzzy controller. A single sensor is used for feedback, and the actuator is represented by altering the boundary conditions of the Ginzburg-Landau model. The closed-loop feedback strategies examined included P control, PID control and fuzzy logic control. Results show that the fuzzy logic control provided the best response with respect to the onset of vortex suppression, the settling times, as well as the control effort applied.

The results indicate that for a single sensor scheme, the increase in the sophistication of the control results in significantly shorter settling times. However, there is only a marginal improvement concerning the suppression of the wake at higher “Reynolds numbers”. It seems that the only way to improve this is to incorporate a multi-sensor strategy. This finding clearly substantiates the work done by Gillies[3,5]. He showed that, using proportional control alone, a direct feedback two-sensor strategy stabilizes the Ginzburg-Landau wake up to 12.5% above the critical value as opposed to just 5% using a single sensor strategy.

In addition, a high-dimensional model used to solve the Ginzburg-Landau equations does not appear feasible for real time estimation and control. The reason is that modern control techniques have difficulty in handling a non-linear system having 300 degrees of freedom. Furthermore, the pursuit of a low-dimensional model may result in a more effective closed-loop control strategy. Wake flows, represented by the Ginzburg-Landau model, are dominated by the dynamics of a relatively small number of characteristic large-scale spatial structures, as observed in experimental results for a forced vortex street. A desirable controller will on the one hand simply measure and control a *finite number of large-scale spatial structures*. On the other hand, it will keep the complexity of the wake flow *low* by not exciting it into a higher dimensional state. If the complex spatio-temporal information is characterized by a relatively small number of quantities, then feedback can be computationally feasible. Therefore, to obtain a controller that can be implemented, a reduced-order model is sought to represent the characteristic features of the flow field.

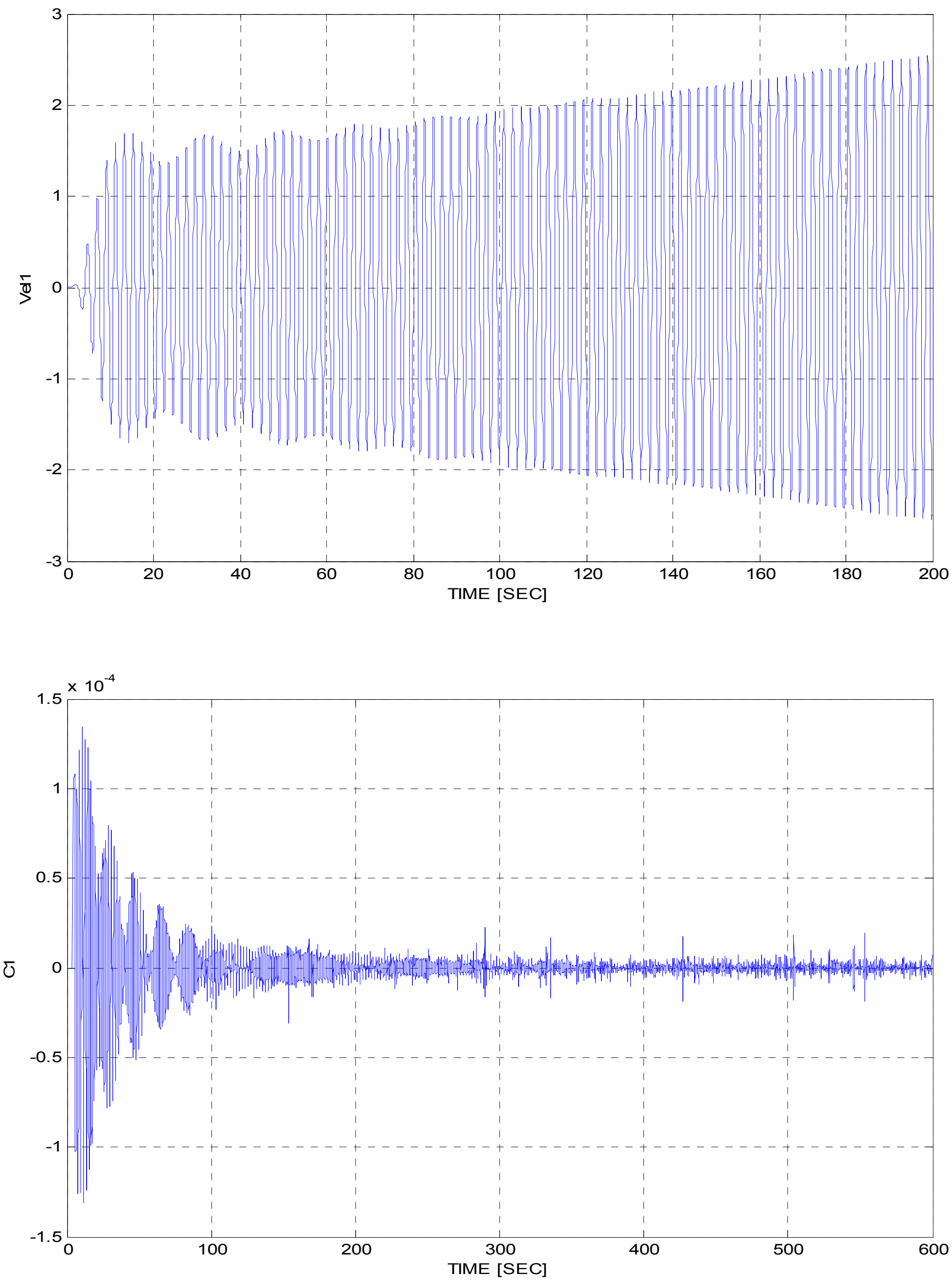
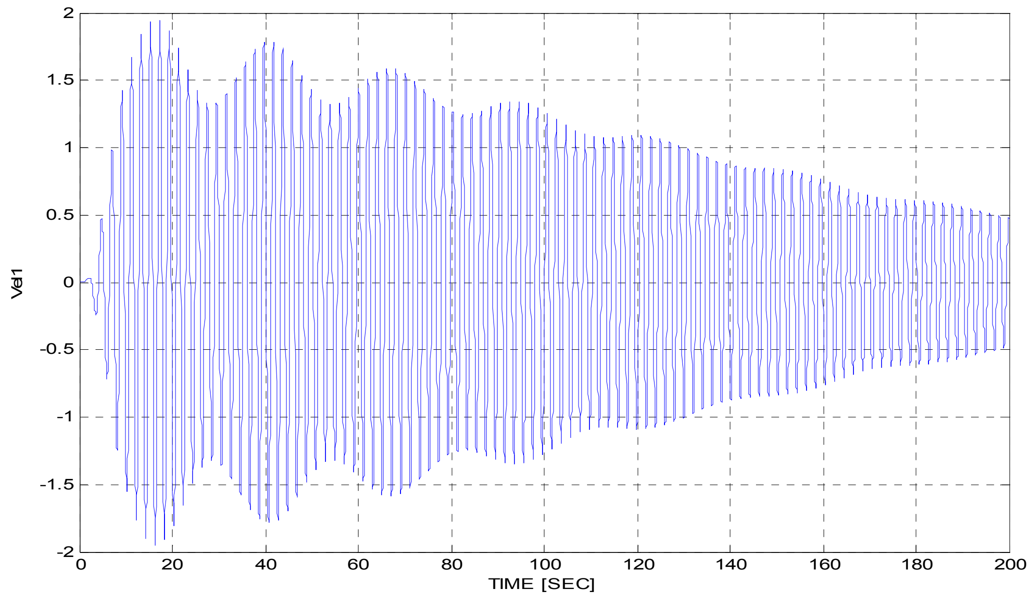


Fig. 4: Time History of Re(A(10,t)) (top figure) and the Control Input (bottom figure) for the PID Controller ($\mu_o = 3.578$)



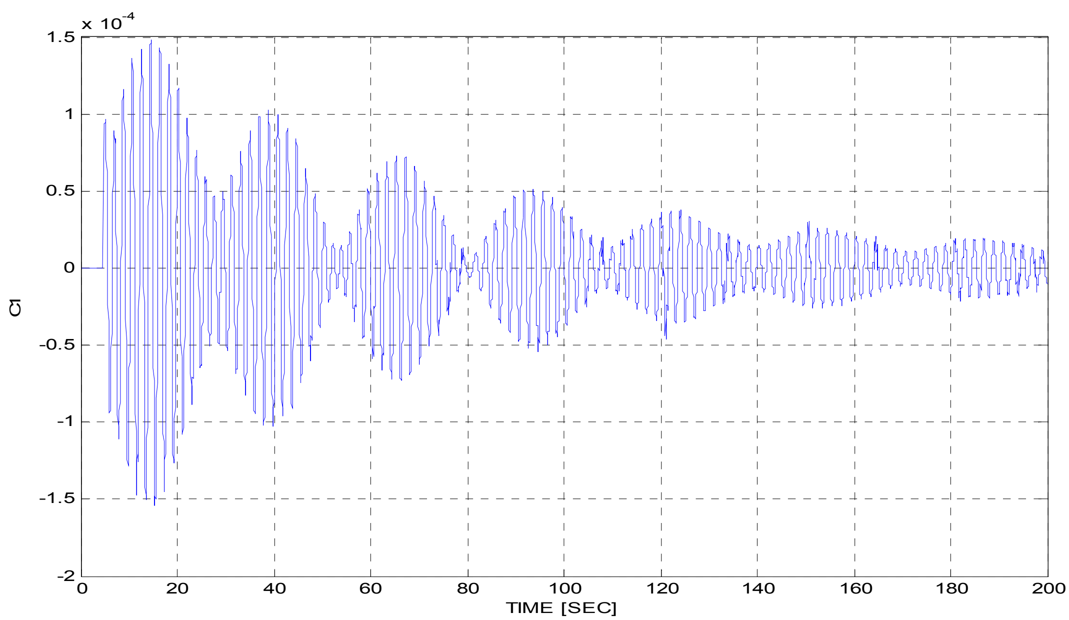


Fig. 5: Time History of Re(A(10,t)) (top figure) and the Control Input (bottom figure) for the Fuzzy Controller ($\mu_o = 3.578$)

Further work includes the development of a low-dimensional model based on Proper Orthogonal Decomposition. The placement and number of sensors will then be determined by the number of modes of this low-dimensional model. Based on this approach, it will be interesting to find the critical Reynolds number for a two-sensor strategy and compare it with the results obtained using direct feedback by Gillies[3,5]. Finally, the strategy will be tested against the high-dimensional, nonlinear model in closed-loop studies.

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