PROPER ORTHOGONAL DECOMPOSITION MODELING OF A CONTROLLLED GINZBURG-LANDAU CYLINDER WAKE MODEL

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A short computational program was undertaken to evaluate the effectiveness of a closed-loop control strategy for the stabilization of an unstable bluff-body flow. In this effort, the nonlinear one-dimensional Ginzburg-Landau wake model at 20% above the critical Reynolds number was studied. The numerical model, which is a nonlinear partial differential equation with complex coefficients, was solved using the FEMLAB®/MATLAB® package and validated by comparison with published literature. Based on computationally generated data obtained from solving the unforced wake, a low-dimensional model of the wake was developed and evaluated. The low-dimensional model of the unforced Ginzburg-Landau equation captures more than 99.8% of the kinetic energy using just two modes. Two sensors, placed in the absolutely unstable region of the wake, are used for real-time estimation of the first two modes. The estimator was developed using the linear stochastic estimation scheme. Finally, the loop is closed using an PID controller that provides the command input to the variable boundary conditions of the model. This method is relatively simple and easy to implement in a real-time scenario. The control approach, applied to the 300 node FEMLAB® model at 20% above the unforced critical Reynolds number stabilizes the entire wake for a proportional gain of 0.06. While the controller uses only the estimated temporal amplitude of the first mode of Im(A(x,t)), all higher modes are stabilized. This suggests that the higher order modes are caused by a secondary instability that is suppressed once the primary instability is controlled.

Nomenclature

$A(x,t)$	Complex amplitude of the Ginzburg-Landau model
C_{ij}	Coefficients of the linear stochastic estimator
c_n, c_d	Complex coefficients of the Ginzburg-Landau model
$F(x,t)$	External forcing in the Ginzburg-Landau model
$G_s(t)$	Time-Varying gains
K_P	Proportional gain of the controller
$N(t)$	Noise parameter in the Ginzburg-Landau model
Re	Reynolds number
U	Advection speed
x	Spatial coordinate
z_i	Temporal mode amplitude of POD model
δ	Dirac delta function
$\mu(x)$	Wake growth rate parameter
μ_{crit}	Value where the self-excited oscillations begin
μ_0	Analogue of wake Reynolds number. Also referred to in text as “Reynolds number”

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Introduction

The phenomenon of vortex shedding behind bluff bodies has been a subject of extensive research. Many flows of engineering interest produce the phenomenon of vortex shedding and the associated chaotic response. Applications include aircraft and missile aerodynamics, marine structures, underwater acoustics, and civil and wind engineering. The ability to control the wake of a bluff body could be used to reduce drag, increase mixing and heat transfer, and enhance combustion. Flows with absolute instabilities behind bluff bodies, an archetype of which is the cylinder wake, demonstrate self-excited oscillations even when all sources of noise are removed[1]. Above a critical Reynolds number (Re ~ 47), nondimensionalized with respect to freestream speed and cylinder diameter, in the wake of a two-dimensional (2D) cylinder, a significant region of local absolute instability occurs which results in a global flow instability, known as the von Karman vortex street.

The complex Ginzburg-Landau (GL) equation, with suitable coefficients, models well the absolute instability of bluff-body wakes. The one-dimensional (1D) GL equations provide useful insight for the description of global modes for purely 2D shedding where the spatial coordinate in the GL equation coincides with the streamwise direction[2]. The 1D GL equation, which is derivable from the Navier-Stokes equations, can be modeled to contain all of the stability features of the 2D cylinder wake pertinent to control. Furthermore, the GL model is frequently used in the literature for wake control studies and has been shown to allow semi-qualitative predictions of the wake with feedback[3-5]. An attractive characteristic of the GL model is that it is relatively straightforward to integrate numerically, making it an effective tool for investigating prototypes of control strategies.

Cylinder wake flows, represented by the GL model, are dominated by the dynamics of a relatively small number of characteristic large-scale spatial structures, as observed in experimental results for periodically forced vortex streets. The GL model is a set of complex, non-linear partial differential equations using numerical finite element or difference schemes. A control model based on these equations is therefore not feasible for real time estimation and control. A desirable controller will on the one hand simply measure and control a *finite number of large-scale spatial structures*. On the other hand, it will keep the number of modes of the wake flow *low* by not exciting it into a higher dimensional state.

If the complex spatio-temporal information is characterized by a relatively small number of quantities, then feedback may be computationally feasible. Therefore, to obtain a controller that can be implemented, a reduced-order model is sought to represent the characteristic features of the flow field.

A common method used to reduce the model-order is proper orthogonal decomposition, commonly known as POD. The POD method may be used to identify the characteristic features, or modes, of a cylinder wake as demonstrated by Gillies[1]. This method is an optimal approach in that it will capture a larger amount of the flow energy with fewer modes than any other decomposition of the flow[6]. Low-dimensional modeling, based on POD techniques, is a vital building block when it comes to realizing a structured model-based closed-loop strategy for flow control. The major building blocks of this structured approach are comprised of a reduced order POD model, a state estimator and a controller. A truth model, based on numerical techniques, is required for computational simulations to verify the effectiveness of the developed approach. Finally, wind/water tunnel experimentation would be required for experimental demonstration.

In his investigations of active control schemes for stabilizing the Ginzburg-Landau wake, Gillies[3,4] showed that a proportional control strategy using a single sensor can successfully control the flow at no more than 5% above criticality. However, a proportional controller based on two sensors extends the envelope to 12% above criticality. At this point, Gillies suggested that more sensors be introduced for feedback purposes for a further extension of the criticality envelope. In this effort, a low-dimensional POD model is sought to control the wake of the Ginzburg-Landau model studied by Gillies[3,4] and Park, Ladd and Hendricks[5]. The introduction of a POD model provides a more effective method of controlling the Ginzburg-Landau wake based on two sensors. The paper is organized as follows: The next section describes the research objective and uniqueness of the developed approach. The Ginzburg-Landau equation is presented in the following section and the FEMLAB® model is described subsequently. Then, the open-loop low-dimensional POD model is presented, followed by the development of the estimation and controller schemes. A comparison of the results between the closed-loop simulations to that in literature is made in the penultimate section, and the conclusions to date of this research effort are summarized in final section.

Research Objective

Recent research on closed-loop control of the Ginzburg-Landau model using a simple proportional fixed gain approach includes work by Park, Ladd and Hendricks[5], Roussopoulos and Monkewitz[2] and Gillies[3,4]. The closed-loop results, obtained using the proportional fixed gain method based on feedback from one or two sensors, provide only limited improvement concerning the extension of the 'vortex shedding' criticality. Gillies[4] recommended the introduction of multiple sensors for a further extension of criticality.

The main objective of this research effort is to develop an effective estimation and control scheme for closed-loop suppression of the Ginzburg-Landau model of the von Karman vortex street. The developed strategy would aim at extending criticality without necessarily having to increase the number of sensors above two. Furthermore, the estimator would be designed to adapt to changes in Reynolds Number for a fixed set of sensor placement and number.

The Ginzburg-Landau Wake Model

The 1D Ginzburg-Landau (GL) equation chosen is based on Gillies (2000) as follows:

$$\frac{\partial A}{\partial t} + U\frac{\partial A}{\partial x} = \mu(x)A + (1 + jc_d)\frac{\partial^2 A}{\partial x^2} - (1 + jc_n)|A|^2 A + F(x,t) \quad (1)$$

where A(x,t) is the complex amplitude and U, c_d, c_n and $\mu(x)$ are real. F(x,t) incorporates the effects of feedback actuation and noise. The stability of the GL "wake" is defined by the growth parameter

$$\mu(x) = \mu_o + \mu' x \quad (2)$$

where μ_o is similar to a Reynolds number based on the cylinder diameter. For $\mu' < 0$ the stability features of this "prototype" wake are similar to the stability features of a 2D cylinder wake; i.e. a self-excited unstable response emerges followed by a limit cycle (see Fig. 1).

For the flow around cylinders, several forcing techniques affect the behavior of the flow; however, the wake response to forcing is similar for each, whether translation of the cylinder in the direction parallel to or perpendicular to the mean flow, rotation of the cylinder or alternate blowing and suction at the separation points[3]. Controlled forcing of the wake will be introduced into the GL equation by an actuation function placed in the near wake, namely F(x,t), using simple delta functions. The actuator will provide a step perturbation to the complex amplitude over the spatial actuation range: $0 < x_a < 2.0$.

The effects of feedback and noise may be incorporated into the GL equation in two ways. The first method, suggested by Roussopoulos and Monkewitz (1996) and Gillies (2000), involves modeling of the actuator as a delta function that provides forcing at a fixed spatial location, x_a, and a delta function that provides sensing at a sensor location, x_s, as shown in Equation (3):

$$F(x,t) = \sum_{s=1}^{n} [G_s(t) A(x_s, t)].[\delta(x - x_a)] + N(t)\delta(x - x_a) \quad (3)$$

The signals from n sensors are fed back with time-varying gains $G_s(t)$. The second term, N(t), describes the noise added to the system. Noise may be modeled with a random number generator of adjustable amplitude. An alternative method for incorporating feedback and noise, utilized in this study, was proposed by Park, Ladd and Hendricks (1993). This approach involves the modeling of an active boundary condition at x = 0 for the Ginzburg-Landau equation as follows:

$$A(0, t) = \alpha(t) A(x_s, t) \quad (4)$$

where α (t) may be time-invariant as in the case of Park, Ladd and Hendricks (1993) and Gillies (2000) or it may be a variable gain. The current effort is based on Gillies' (2000) model to enable comparison:

- 1D domain $0 < x < 120$
- Boundary conditions: A(0,t) = 0 (which simulates the cylinder body), A(120,t) = 0
- Fixed Parameters: U = 5; μ'= -0.0434; c_d = 1; c_n = 0.

Computational Model

After writing the GL equation, which is a non-linear partial differential equation with complex coefficients, the next step is to solve it numerically. After a survey of the market for an appropriate solver, FEMLAB®, developed by COMSOL[10], was selected. FEMLAB® is an interactive environment for finite-element modeling and simulating scientific and engineering problems based on partial differential equations (PDEs). FEMLAB's® ability to arbitrarily define and couple any number of nonlinear PDEs, as well as work within the MATLAB®/SIMULINK® environment, makes it an attractive tool for studying fluid-control interaction. Furthermore, the solution of the Ginzburg-Landau equation is provided by FEMLAB® as a benchmark in their model library[10]. The size of finite element model was determined by trial and error and 300 elements provided a fairly accurate model. The feedback forcing is introduced into the model by perturbing the boundary conditions at x = 0.

The details of the FEMLAB® model of the Ginzburg-Landau equation are as follows:

- **Element Type:** 'solid1(x)' – creates a 1-D solid object that spans all the coordinate values in the vector x (1-D domain $0 < x < 120$)
- **Number of Elements:** 300
- **Number of Nodes:** 301
- **Boundary Conditions:** A(0,t)=0; A(120,t)=0
- **Time-Step:** 0.2 units
- **Total Run-time:** 60 units
- **Initial condition:** A(x,0)=0.0001

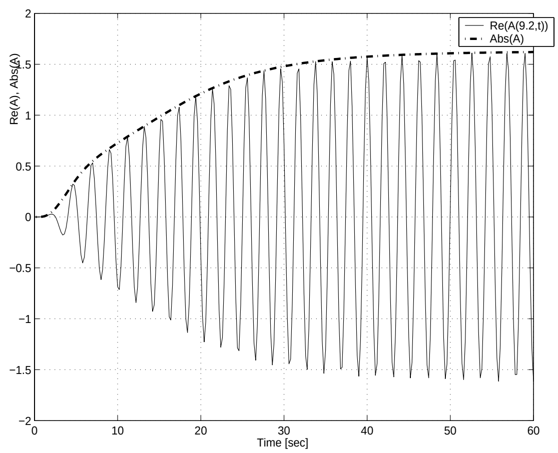

Figure 1: Temporal plot of real part (solid line) and modulus (dot-dashed line) of A(9.2, t) at 5% above μ_{crit}

Gillies[3] reported that his wake model exhibits self-excited wake oscillations above $\mu_o = \mu_{crit} = 3.43$ The above coefficients for the study of the spatially developing flows, based on the Ginzburg-Landau model, were first introduced by Park, Ladd and Hendricks[5]. The value of μ_{crit} is of importance; therefore an important step for validating the current work was to arrive at the same value of μ_{crit} using the model currently developed (using FEMLAB®) as Gillies[3]. Simulation results show that the FEMLAB model predicts the value of μ_{crit} to within 0.3% of that obtained by Gillies[3] based on the same coefficients of the Ginzburg-Landau equation. Figure 1 displays the temporal plot of the real part of A(x, t) at 5% above criticality. Initially, the amplitude of the real part of A(x, t) grows exponentially, and then it almost equilibrates at a saturated level, or limit cycle, due to the stabilizing cubic nonlinearity. Furthermore, the FEMLAB® results shown in Fig. 1 compare well with those presented by Gillies[3]. Following the successful modeling of the open-loop behavior of the Ginzburg-Landau equation using FEMLAB®, the model may be exported to SIMULINK® for closed loop studies. In this effort, two conditions are examined for the closed-loop studies, namely, 12.5% above μ_{crit} and 20% above μ_{crit}.

Proper Orthogonal Decomposition

Feasible real time estimation and control of the GL model may be effectively realized by reducing the model complexity using POD techniques. POD, a non-linear model reduction approach referred to in the literature as the Karhunen-Loeve expansion[6] is based on the spectral theory of compact, self-adjoint operators. The desired POD model contains an adequate number of modes to enable reasonable modeling of the temporal and spatial characteristics of the large-scale coherent structures inherent in the flow. Further details of the POD modeling may be found in Graham, Peraire and Tang[7,8].

In this effort, the method of "snapshots" introduced by Sirovich[9] is employed to generate the basis functions of the POD spatial modes from the numerical solution of the GL equations obtained using FEMLAB®. The resulting spatial modes of the POD enable the GL equations to be projected using a least-squares method to yield a set of ordinary differential equations (O.D.E.). The POD algorithm was realized in MATLAB and contains the following steps:

Step I - Load and arrange data obtained from the FEMLAB® solution of the GL wake model.

Step II - Adjust the data so that the mean of the ensemble of snapshots, represented by vectors, v, is zero. This is accomplished by computing the 'average snapshot' and then subtracting this profile from each member of the ensemble. This is done mainly for reasons of scale; *i.e.* the deviations from the mean contain information of interest but may be small compared with the original signal.

Step III - Compute the empirical correlation matrix, R. A simple approximation technique is applied to obtain the numerical integration. In this effort, the correlation matrix is computed using the inner product.

Step IV - Compute the eigenvalues and the eigenfunctions. Since the eigenvalues measure the relative energy of the system dynamics contained in that particular mode, they may be normalized to correspond to a percentage.

Step V – Check the orthogonality with the Kronecker delta function for the orthogonality matrix of the eigenfunctions.

Step VI – Determine the time histories of the temporal coefficients of the POD model by applying the least squares technique to the spatial modes and the unforced flow.

Step VII – If required, reconstruct the system response based on the low-dimensional model.

The POD algorithms, based on the above steps and realized in MATLAB®, were applied to the two conditions studied, namely, 12.5% above μ_{crit} and 20% above μ_{crit}. The energy content for the first four modes of the real and the imaginary parts of A(x,t) is presented in Table 1. It can be seen that more than 99.8% of the kinetic energy of the flow lies in the first two modes. It will be shown later that the estimation process is based on the Im(A(x,t)) alone.

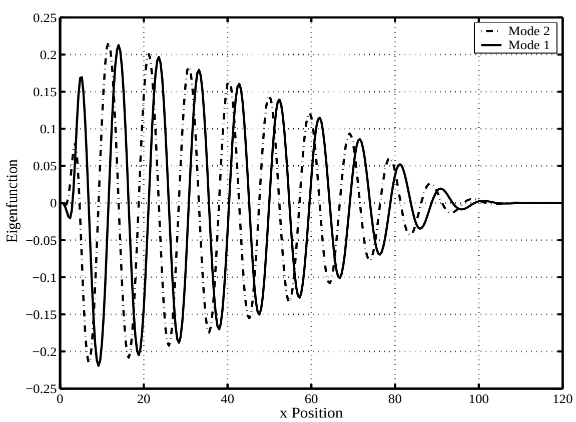

Figure 2: Normalized Spatial Eigenfunctions of Im(A(x,t)) at 12.5% above μ_{crit}

The mode shapes for the first two modes for Im(A(x,t)) are presented in Figures 2 and 3 for both the conditions described in Table 1.

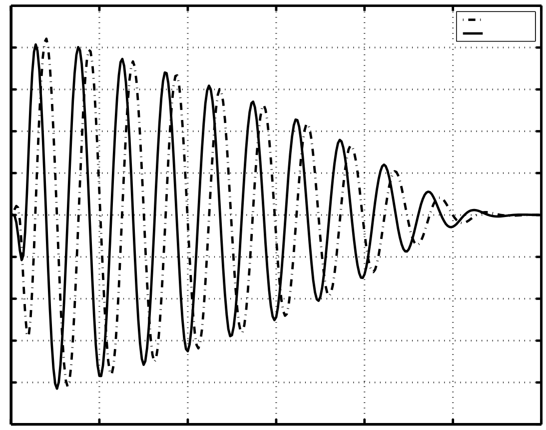

Figure 3: Normalized Spatial Eigenfunctions of Im(A(x,t)) at 20% above μ_{crit}

A close look at Figures 2-3 shows the effect of increasing "Reynolds Number" on the mode shapes. As we move from 12.5% above μ_{crit} (Figure 2) to 20% above μ_{crit} (Figure 3), we observe that there is an increase in the value of the normalized eigenfunctions between 0-10 along the x-axis. This is the area referred to by Gillies[3] as the "absolutely unstable" region. The multiple sensor strategy examined by Gillies[3] comprised of placing two sensors: one at $x_1 = 4.8$ and the other at $x_2 = 9.6$. The signals received from these two sensors were utilized in the proportional feedback control law. This strategy enabled stabilizing the Ginzburg-Landau wake up to 12.5% above μ_{crit}.

Condition Studied	**Mode I**		**Mode II**		**Mode III**		**Mode IV**	
	Re	Im	Re	Im	Re	Im	Re	Im
12.5% Above Critical	50.868 %	50.716 %	49.015 %	49.168 %	0.063 %	0.063 %	0.054 %	0.053 %
20.0% Above Critical	50.150 %	50.149 %	49.848 %	49.849 %	0.001 %	0.001 %	0.001 %	0.001 %

Table 1: Energy content for the first four modes of the POD model

The estimation scheme developed in the next section takes the variations of the amplitudes of the normalized eigenfunctions into account. Two sensors are located similarly as above, namely one at $x_1 = 4.8$ and the other at $x_2 \sim 9.6$ and an adaptive estimation strategy is developed to account for the changes in the mode shapes. In this effort, the x-axis grid allows positioning of the two sensors at $x_1 = 4.8$ (node 13 of the FEMLAB® model) and at $x_2 = 9.2$ (node 24 of the FEMLAB® model). The implications of this approach make sense. In a realistic application, the sensor locations are usually fixed. The gains of the estimator may adapt to the Reynolds Number, calculated in real-time based on velocity/pressure measurements, by utilizing a look-up table.

Modal Estimation

The time histories of the temporal coefficients of the POD model are determined by applying the least squares technique to the spatial modes and the unforced flow. These time histories are presented in Figures 5 and 6 for both the conditions described in Table 1. The general hypothesis of this research effort is that the controller should be based on estimates of not more than Modes 1 and 2. The motivation is that for practical applications it is desirable to reduce the information required for estimation to the minimum. The requirement for the estimation scheme then is to behave as a modal filter that has "combed out" the higher modes. The main aim of this approach is to thereby circumvent the destabilizing effects of observation spillover as described by Balas[11]. Spillover has been the cause for instability in the control of flexible structures and modal filtering was found to be an effective remedy[12] .

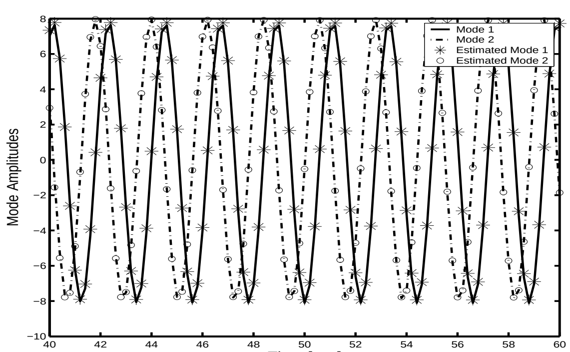

Figure 5: Temporal Mode Amplitudes, z_1 & z_2 , of Im(A(x,t)) at 12.5% above μ_{crit}

The intention of the proposed strategy is that the signals provided by the two sensors placed at $x_1 = 4.8$ and at $x_2 = 9.2$ are processed by the estimator to provide the estimates of the first two modes. The estimation scheme, based on the linear stochastic estimation procedure introduced by Adrian[13], predicts the temporal amplitudes of the first two POD modes from a finite set of measurements obtained from the uncontrolled solution of the Ginzburg-Landau wake model. Further details of stochastic estimation of POD modes are provided by Bonnet et al.[14]. Recently, this method was also used by Carlson and Miller[15] to predict the degree of flow separation from POD modes on a backward facing ramp using ramp pressure measurements.

A set of 100 measurements at $x_1 = 4.8$ and at $x_2 = 9.2$ were extracted from the uncontrolled steady-state FEMLAB® solution. These measurements were at intervals of 0.2 seconds from t = 40 s. until t = 60 s. The temporal mode amplitudes, z_1 and z_2, obtained in the previous section at the above 100 discrete times were mapped onto the extracted sensor signals, $\mathrm{Im}(A(x_1,t))$ and $\mathrm{Im}(A(x_2,t))$, as follows:

$$z_1(t) = C_{11} * \mathrm{Im}(A(x_1,t)) + C_{12} * \mathrm{Im}(A(x_2,t)) \quad (5)$$
$$z_2(t) = C_{21} * \mathrm{Im}(A(x_1,t)) + C_{22} * \mathrm{Im}(A(x_2,t)) \quad (6)$$

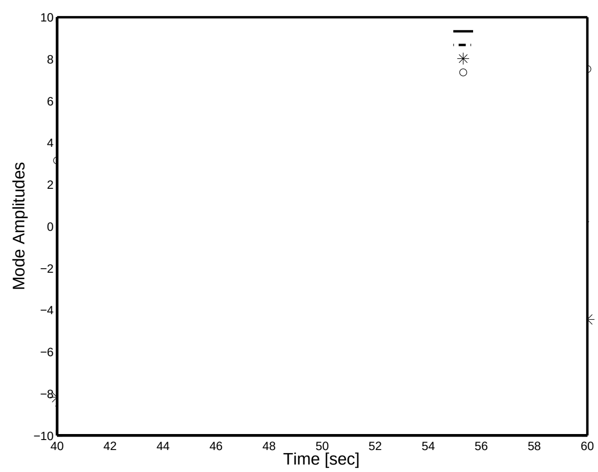

Figure 6: Temporal Mode Amplitudes, z_1 & z_2 , of Im(A(x,t)) at 20% above μ_{crit}

The coefficients C_{ij} (i,j = 1,2) in Equations (5)-(6) are obtained via the linear stochastic estimation method from the set of 100 discrete sensor signals and temporal mode amplitudes and are presented in Table 2. This procedure is done just once for each Reynolds Number before closing the feedback control loop. The main advantage of this approach is that it enables usage of sensors at fixed sensor locations with just the coefficients C_{ij} adapting to the Reynolds Number using the look-up table presented in Table 2. Basically, C_{ij} represents the gains of the estimator and the look-up table is the gain scheduler based on the modal estimation provided by Equations (5)–(6). Figures 5 and 6 illustrate the effectiveness of the linear stochastic estimation process for the estimation of the first two temporal mode amplitudes z_1 and z_2 .

Condition Studied	C_{11}	C_{12}	C_{13}	C_{14}
12.5% Above Critical	1.4524	4.1250	5.1486	-4.1643
20.0% Above Critical	-4.2744	6.9612	-4.7633	-0.1911

Table 2: Coefficients C_{ij} of the modal estimator

The Closed-Loop System

The closed-loop system, realized in MATLAB®/SIMULINK®, is presented in Figure 7. The FEMLAB® subsystem contains the 300 node, non-linear Ginzburg-Landau model. The four signals out of FEMLAB® are as follows: y_1= Re (A(4.8,t)) and y_2 = Re(A(9.2,t)). They are used for purposes of observation alone, whereas, y_3 = Im (A(4.8,t)) and y_4 = Im(A(9.2,t)) are used for the estimation of the temporal mode amplitude. Monitoring y_1 and y_2 as well makes sure that the real part of the wake solution, A(x,t), is stabilized (see Figures 8 and 9). On the other hand, the best indicator that the imaginary part of the solution A(x,t), is stabilized is by observing the control input (see Figure 10). Although, we are capable of providing estimates for both Modes 1 and 2 based on the sensor signals extracted from x_1 = 4.8 and x_2 = 9.2, it was found that for the two conditions studied, it is adequate for the controller to be based on the estimate of Mode 1 alone. In Figure 7, Gain 2 and Gain 3 in the Estimator Block correspond to coefficients C_{11} and C_{12} respectively. The input to the control block is just the estimate of Mode 1. The control scheme is designed such that above a certain threshold for the derivative of estimated Mode 1, a constant control is applied. Below the threshold value a Proportional-Integral-Derivative (PID) controller is applied. A proportional controller was found to be effective enough for the conditions studied i.e. K_D and K_I were zero. A rate limiter, with a slew rate of ± 1.2, was introduced to ensure that physical commands would be introduced by the controller into the FEMLAB subsystem. Finally, the control was activated arbitrarily after ensuring that the system had reached its limit cycle. The control parameters used for the two conditions studied are provided in Table 3, which is in fact a gain-scheduling look-up table that enables the controller to adapt to a varying Reynolds Number.

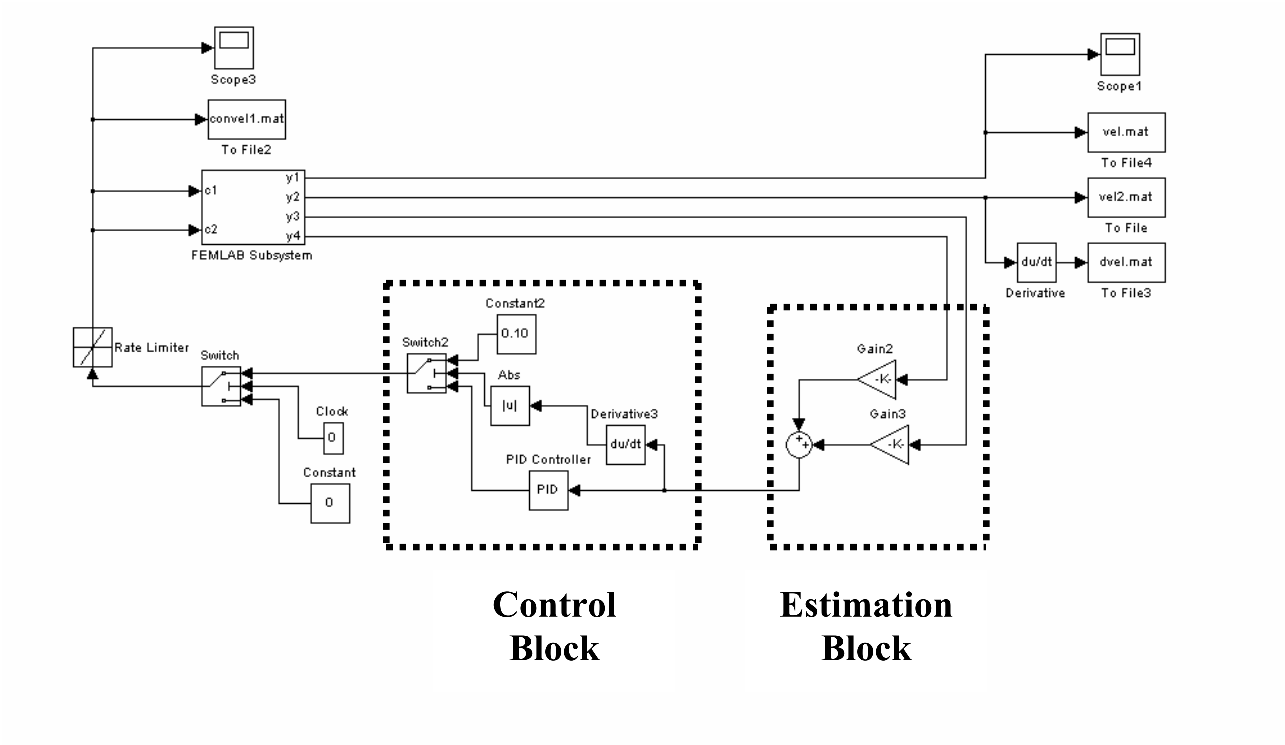

Figure 7: SIMULINK Model of the Closed-Loop Control System

Condition Studied	K_P	Constant Control	Derivative Threshold	Rate Limiter Slew Rate	Controller Switch-On Time [sec]
12.5% Above Critical	0.017	0.10	5.0	± 1.2	35
20.0% Above Critical	0.060	0.75	4.0	± 1.2	30

Table 3: Parameters of the Controller for the two conditions studied

Simulation Results

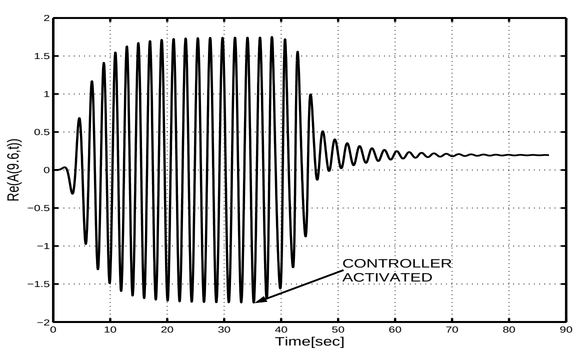

Figure 8: Controlled Wake Signal at 12.5% Above Critical

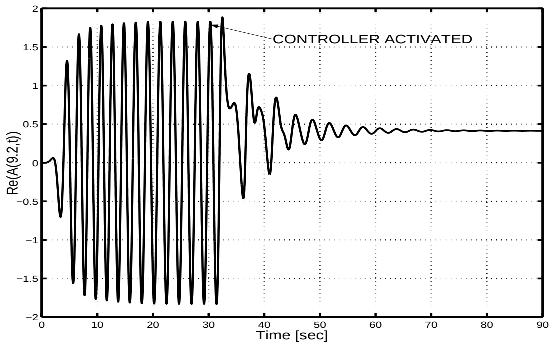

Figure 9: Controlled Wake Signal at 20% above Critical

The results of the simulation are provided in Figures 8-10. The estimation/control scheme developed in this effort provides effective stabilization of the Ginzburg-landau wake at the two conditions studied. The non-linear dynamics of the Ginzburg-Landau model has a fixed point that is not at $A(x,t) = 0$, and the controller stabilizes the wake by converging to this fixed point (see Figures 8-9). The control input time-history, shown in Figure 10, illustrates the behavior of the controller for large values of the estimated derivative of Mode 1 ($25 < t < 40$). After 40 seconds, the PID controller takes over completely and stabilizes the system. The rate limiter ensures that the control command remains physically realizable in nature.

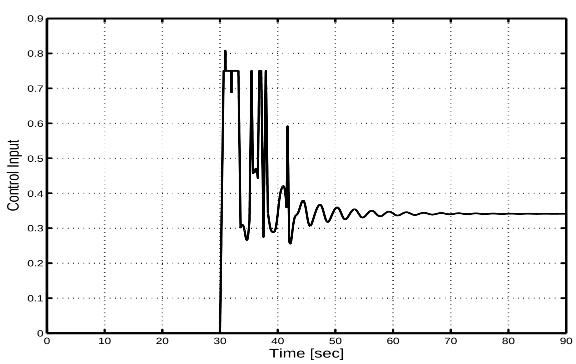

Figure 10: Control Input at 20% Above Critical

Conclusions

A FEMLAB® model was developed to solve the Ginzburg-Landau equation that contains all the stability features of the 2-D cylinder wake pertinent to control. The developed model was exported to SIMULINK® for closed-loop studies. A gain-scheduling approach is proposed, whereby, for a given sensor configuration, the estimation and controller gains adapt by means of a look-up table to the variation in the Reynolds Number. This method is relatively simple and easy to implement in a real-time scenario. The control approach, applied to the 300 node FEMLAB® model at 20% above the unforced critical Reynolds number stabilizes the entire wake for a proportional gain of 0.06. While the controller uses only the estimated temporal amplitude of the first mode of $Im(A(x,t))$, all higher modes are stabilized. This suggests that the higher order modes are caused by a secondary instability. Thus they are suppressed once the primary instability is controlled.

As opposed to the recommendations proposed by Gillies[3], who recommended the introduction of more sensors, it has been shown that by introducing an estimation scheme based on a low-dimensional POD model, two sensors are adequate for a substantial further extension of the onset of the instability from 12.5% above criticality to 20% above criticality. Continuing studies will aim at extending the onset of instability still further in an attempt to examine the limits of a two-sensor estimation/control strategy. In addition, the control approach will be further examined to observe its sensitivity to time delays, actuator limitations, modeling and estimation errors and sensor noise. In parallel, this approach will be applied to experimental and computational studies of the control of a two-dimensional circular cylinder wake at the US Air Force Academy.

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