SENSOR PLACEMENT BASED ON PROPER ORTHOGONAL DECOMPOSITION MODELING OF A CYLINDER WAKE

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I. INTRODUCTION

The phenomenon of vortex shedding behind bluff bodies has been a subject of extensive research. Many flows of engineering interest produce the phenomena of vortex shedding and the associated chaotic response. Applications include aircraft and missile aerodynamics, marine structures, underwater acoustics, civil and wind engineering. The ability to control the wake of a bluff body could be used to reduce drag, increase mixing and heat transfer, and enhance combustion. Flows with absolute instabilities behind bluff bodies, an archetype of which is the cylinder wake, demonstrate self-excited oscillations even when all sources of noise are removed (Gillies, 1998). Above a critical Reynolds number (Re ~ 47), nondimensionalized with respect to freestream speed and cylinder diameter, in the wake of a two-dimensional (2D) cylinder, a significant region of local absolute instability occurs which results in a global flow instability, known as the Karman vortex street.

Cylinder wake flows, governed by the Navier-Stokes Equations, are dominated by the dynamics of a relatively small number of characteristic large-scale spatial structures, as observed in experimental periodically forced vortex sheets. A control model based on these equations is therefore not feasible for real time estimation and control. A desirable controller will on the one hand simply measure and control a *finite number of large-scale spatial structures*. On the other hand, it will keep the dimension of the wake flow *low* by not exciting it into a higher dimensional state causing higher order modes to be generated. If the complex spatio-temporal information is characterized by a relatively small number of spatial structures or modes, then feedback can be computationally feasible. Therefore, to obtain a controller that can be implemented, a reduced-order model is sought to represent the salient features of the flow field.

A common method used to reduce the model-order is proper orthogonal decomposition, commonly known as POD. The POD method may be used to identify the characteristic features, or modes, of a cylinder wake as demonstrated by Gillies (1998). This method is an optimal approach in that it will capture a larger amount of the flow energy in fewer modes than any other decomposition of the flow (Holmes, Lumley, and Bekooz, 1996). Low-dimensional modeling, based on POD techniques, is a vital building block when it comes to realizing a structured model-based closed-loop strategy for flow control. The major building blocks of this structured approach are presented in Fig. 1 and are comprised of a reduced order POD model, a state estimator and a controller. A truth model, based on numerical techniques, is required for computational simulations to verify the effectiveness of the developed approach. Finally, wind/water tunnel experimentation would be required for experimental demonstration.

In this effort, a POD model is sought to represent the CFD (Computational Fluid Dynamics) solution to the two-dimensional Navier-Stokes equations in a manner suitable for real-time feedback control. The POD model is verified using experimental water tunnel data. The main objective is to develop a scheme for effective sensor placement and number based on the POD model. The effectiveness of this scheme will be evaluated with data obtained from CFD simulations and from water tunnel experiments. The paper is organized as follows: Section II describes the research objective. The CFD model is presented in Section III, and the water tunnel model is described in Section IV. The open-loop POD model is developed in Section V. The sensor number and placement scheme will be developed in Section VI followed by a comparison of the results between the CFD simulations and the experimental data. Finally, the conclusions to date of this research effort are summarized in Section VII.

II. RESEARCH OBJECTIVE

Recent research on closed-loop control of the wake instabilities by Park, Ladd and Hendricks (1993) and Gillies (2001) has addressed the issue of sensor placement and number. The method used has been based on trial-and-error with a new search cycle required for each Reynolds number. In addition, direct feedback control of the sensor signals has not proven to be very effective in controlling wakes and the subsequent closed-loop results, obtained using the proportional fixed gain method, provide only marginal improvement. To pursue a more effective closed-loop control strategy based on a low-dimensional POD model, it is imperative to develop a structured search pattern for sensor placement and number. The main objective of this research effort is to develop such a scheme and to evaluate it using CFD simulations as well as water tunnel data.

III. THE CFD MODEL

The CFD model is based in the Cobalt flow solver, developed and distributed by COBALT solutions, with a two-dimensional, second order, finite volume unstructured grid. The simulation runs on USAFA's Beowulf Linux Cluster (32 Nodes / 64 Processor P3/1GHz). A unique feature of this simulation is that it has been modified to incorporate custom feedback control. This is accomplished by writing sensor information after calculating one time step to a interface file. Then, the simulation "waits" for a control algorithm to update the file with an actuator position command. The command is fed back to the simulation before calculating the next time step. This way any arbitrary control algorithm may be implemented using the MATLAB/SIMULINK package running on a separate PC independent of the CFD solver. Open-loop CFD results for Re = 120 are presented in Fig. 2. The final paper will include a detailed description of the CFD model.

IV. EXPERIMENTAL MODEL

For an experiment involving closed loop feedback flow control of a arman vortex street behind a circular cylinder, real time flow field information at several locations in the wake downstream of the cylinder was needed. This experiment was to be conducted in a low speed water tunnel in order to achieve the required Reynolds number, and a low vortex shedding frequency. Target parameters are a Reynolds number of 120, at which the water tunnel flow will show a vortex shedding frequency of 1.2 Hz. To obtain enough instantaneous information on the flow field, several sensor locations in the wake need to be monitored. Additionally, the flow field features an absolute instability downstream of the cylinder. This region is extremely sensitive to intrusion by sensors, at the same time it provides the most valuable flow field information. Therefore non-intrusiveness of the sensors is an important requirement. An in-house unique real-time Particle Image Velocimetry (PIV) system was developed to enable closed-loop studies. The experimental set-up is detailed in Siegel et al (2003). The closed-loop system commands the two voice coil actuators that translates the cylinder orthogonal to the flow direction. A schematic model of the experimental model is provided in Fig. 3. The salient features of this model are:

- Cylinder Model: D = 3.97 mm
- Span: L = 381 mm
- Aspect Ratio: L/D ~ 95
- Shedding Frequency: 1.22 Hz
- Vertical Travel: +/- 4 mm
- Actuator Bandwidth: Above 50Hz

The final paper will elaborate on the details of the experimental model.

V. OPEN-LOOP POD MODEL

Feasible real time estimation and control of the cylinder wake may be effectively realized by reducing the model complexity using POD techniques. POD, a non-linear model reduction approach referred to in the literature as the Karhunen-Loeve expansion (Holmes, Lumley, and Bekooz, 1996), is based on the spectral theory of compact, self-adjoint operators. The desired POD model contains an adequate number of modes to enable reasonable modeling of the temporal and spatial characteristics of the large scale coherent structures inherent in the flow. Further details of the POD modeling may be found in Holmes, Lumley, and Bekooz (1996). In this effort, the method of "snapshots" introduced by Sirovich (1987) is employed to generate the basis functions of the POD spatial modes from the CFD solution as well as form experimental data. Recently, Smith, Siegel and Mclaughlin (2002), applied the "snapshot" method to extract a low-dimensional POD model from experimental water tunnel data at Re=120.

In this effort, the POD algorithm was realized in MATLAB and contains the following steps:

Step I - Load and arrange data obtained from the CFD solution/experiment.

Step II - Adjust the data so that the mean of the ensemble of snapshots, represented by vectors, v, is zero. This is accomplished by computing the 'average snapshot' and then subtracting this profile from each member of the ensemble. This is done mainly for reasons of scale; *i.e.* the deviations from the mean contain information of interest but may be small compared with the original signal.

Step III - Computation of the empirical correlation matrix, R. A simple approximation technique is applied to obtain the numerical integration. In this effort, the correlation matrix is computed using the inner product.

Step IV - Computation of the eigenvalues and the eigenfunctions. Since the eigenvalues measure the relative energy of the system dynamics contained in that particular mode, they may be normalized to correspond to a percentage.

Step V - The orthogonality check will obtain the Kronecker delta function for the orthogonality matrix of the eigenfunctions.

Step VI - Transform the original high-dimensional problem to a low-dimensional model merely requiring the solution of a set of N(modes) order O.D.E. using a least squares method.

The first two spatial POD modes extracted from CFD Data at Re = 100 are presented in Fig. 4. The final paper will include a detailed description of the POD model.

VI. SENSOR PLACEMENT AND NUMBER

The time histories of the temporal coefficients of the POD model are determined from applying the least squares technique to the spatial modes and the unforced flow. Currently, the general hypothesis of this research effort at the USAF Academy is that the controller should be based on estimates of not more than Modes 1 and 2. The motivation is that for practical applications it is desirable to reduce the required information for estimation to the minimum. However, to examine the sensitivity of the estimation process, the first four modes will be estimated. The requirement from the estimation scheme is then to behave as a modal filter that has "combed out" the higher modes. The main aim of this approach is to thereby circumvent the destabilizing effects of observation spillover as described by Balas (1978). Spillover has been the cause for instability in the control of flexible structures and modal filtering was found to be an effective remedy (Meirovich, 1990). The intention of the proposed strategy is that the signals provided by the sensors are processed by the estimator to provide the estimates of the first four modes. The estimation scheme, based on the linear stochastic estimation procedure introduced by Adrian (1977), predicts the temporal mode amplitudes of the first four POD modes from a finite set of measurements obtained from the uncontrolled CFD solution / experimental data. Further details of stochastic estimation of POD modes are provided by Bonnet et al. (1994).

Close examination of the spatial modes presented in Fig. 4 point to a topology, whereby the maxima and minima of the three dimensional surface indicate the areas of maximum modal activity. Therefore, sensors placed at these points would be the best observers of the modal activity. So the basic scheme for sensor placement would be to place sensors on a couple of these points per mode and to place more sensors for better the accuracy. However, we would like to obtain a minimum set of sensors for the estimation of all four modes to a desired accuracy level. Fig. 5 illustrates the positioning of 5 sensors using the above heuristics. The temporal mode amplitudes and their estimates using 5 sensors are presented in Fig. 6, indicating excellent agreement. The final paper will contain a detailed analysis of several configurations of sensor placement and number for Reynolds numbers of 100, 120 and 140. Results using CFD data will be compared with those obtained from experimental data.

VII. CONCLUSIONS

A CFD simulation was developed to model a circular cylinder that contains all the stability features of the 2-D cylinder wake pertinent to control. The developed model was verified with data obtained from the water tunnel. For both approaches (CFD and experiment), the spatial modes of the low-dimensional POD model have been extracted. A linear stochastic estimator was used to examine sensor location and number at Re = 100. The final paper will include detailed results and a comparison of results obtained from CFD simulations with experimental data at Reynolds numbers of 100, 120 and 140.

VIII. REFERENCES

Adrian, R.J., "On the role of conditional averages in turbulence theory", Proceedings of the Fourth Biennial Symposium on Turbulence in Liquids, J. Zakin and G. Patterson (Eds.), Science Press, Princeton, 1977, pp. 323-332.

Balas, M.J., "Active Control of Flexible Systems", Journal of Optimization Theory and Applications, Vol. 25, No. 3, July 1978, pp. 217-236.

Bonnet, J.P., Cole, D.R., Delville, J., Glauser, M.N., Ukeiley, L.S., "Stochastic Estimation and Proper Orthogonal Decomposition": Complementary Techniques for Identifying Structure", Experiments in Fluids 17 (1994), pp. 307-314.

FEMLAB, Version 2.2, COMSOL AB., November 2001 (for further information URL: http://www.femlab.com).

Gillies, E. A., "Multi Sensor Control of Vortex Shedding", 6th AIAA/CEAS Aeroacoustics Conference, Lahaina, Hawaii, AIAA Paper 2000-1933, June 12-14, 2000.

Gillies, E. A., "Low-dimensional control of the circular cylinder wake", Journal of Fluid Mechanics, Vol. 371, pp. 157-178, 1998.

Holmes, P., Lumley, J.L., and Berkooz, G., "Turbulence, Coherent Structures, Dynamical Systems and Symmetry", Cambridge University Press, Cambridge, Great Britain, 1996.

Park, D.S., Ladd, D.M., and Hendricks, E.W., "Feedback control of a global mode in spatially developing flows", Physics Letters A 182, pp. 244-248, 1993.

Meirovitch, L., "Dynamics and Control of Structures", John Wiley & Sons, Inc., New York, 1990, pp. 313-351

Roussopoulos, K. and Monkewitz, P.A., "Nonlinear Modeling of Vortex Shedding Control in Cylinder Wakes", Physica D 97, pp. 264-273, 1996.

Siegel, S., Cohen, K., McLaughlin, T., and Myatt, J., "Real-Time Particle Image Velocimetry for Closed-Loop Flow Control Studies", 41st Aerospace Sciences Meeting and Exhibit Reno, Nevada 6-9 January 2003, AIAA Paper 2003-0920.

Sirovich, L., "Turbulence and the Dynamics of Coherent Structures Part I: Coherent Structures", Quarterly of Applied Mathematics, Vol. 45, No. 3, Oct. 1987, pp. 561-571.

Smith, D., Siegel, S., and Mclaughlin T., "Modeling of the Wake Behind a Circular Cylinder Undergoing Rotational Oscillation", 1st AIAA Flow Control Conference, 24-26 June 2002, St. Louis, MO, AIAA Paper 2002-3066.

FIGURES

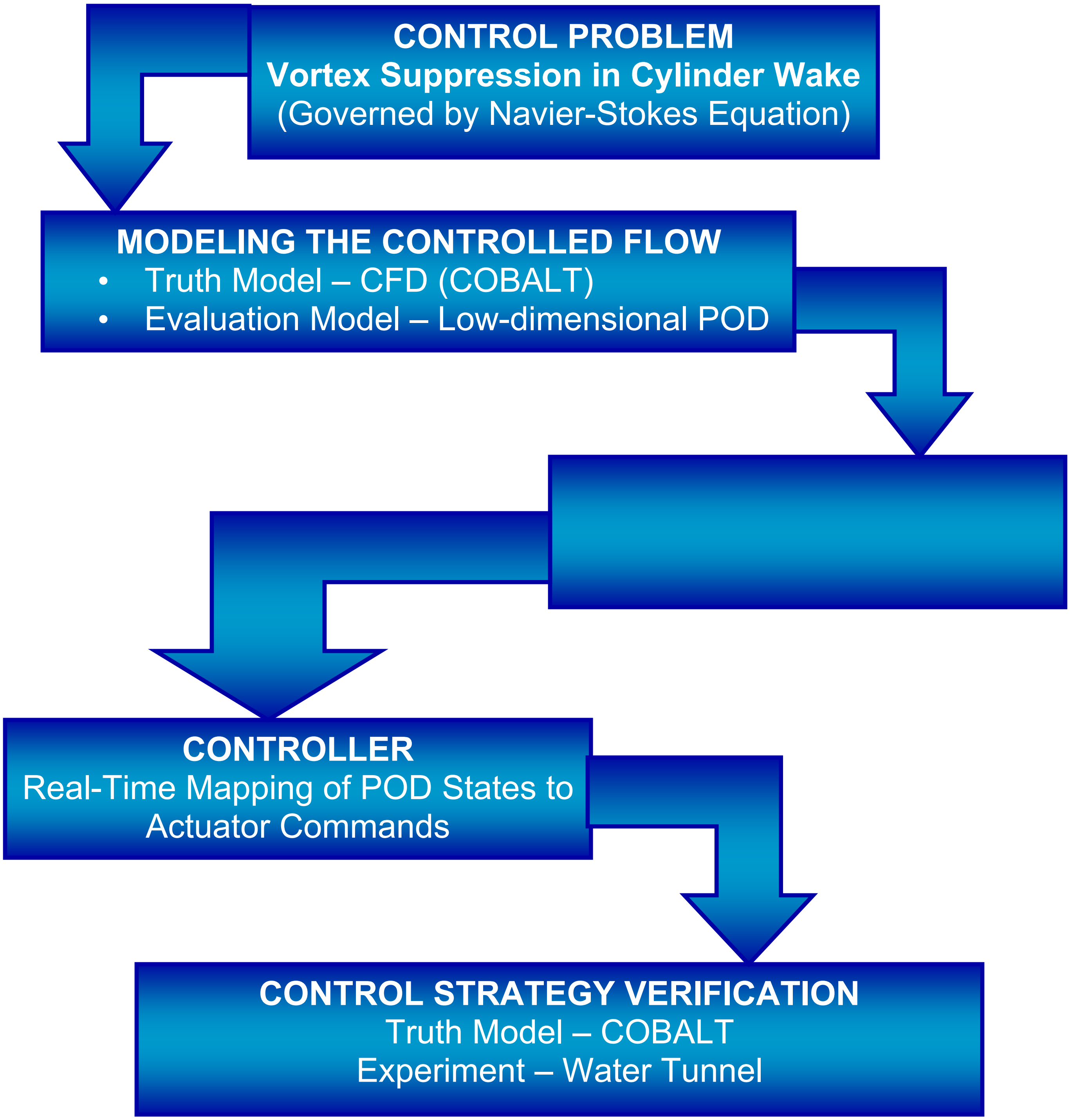

Fig. 1: Building Blocks for Effective Closed-Loop Flow Control

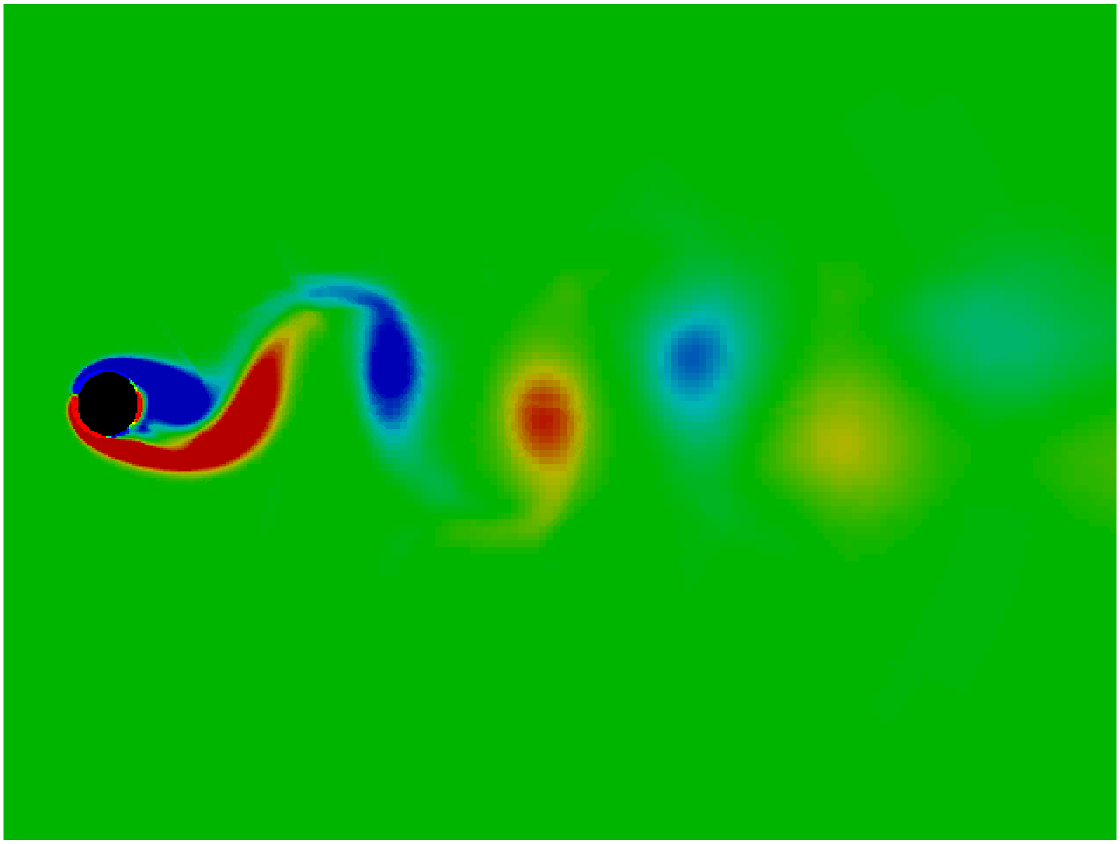

Fig. 2: Open-Loop CFD Results from COBALT at Re = 120

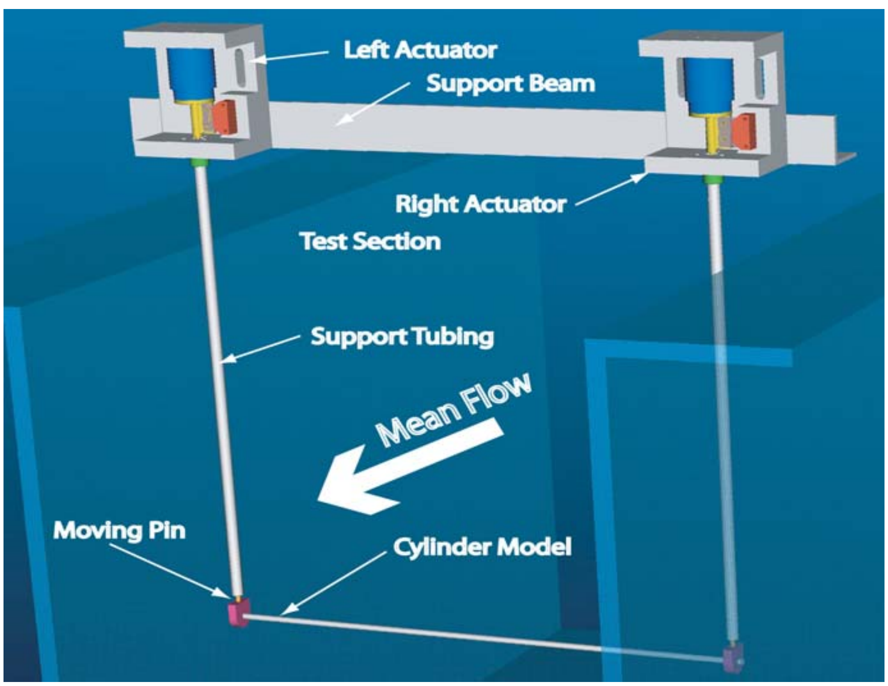

Fig. 3: Experimental Model

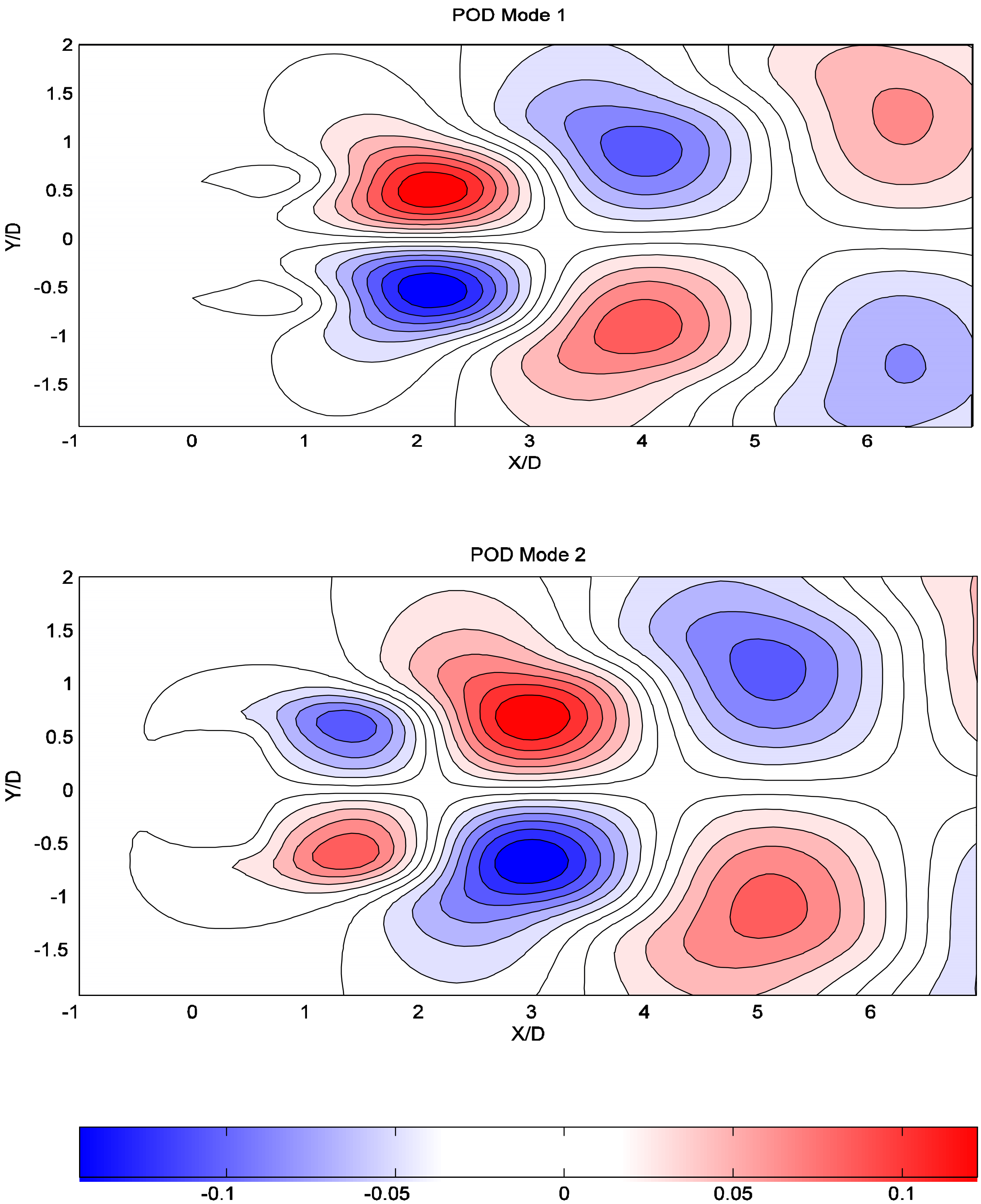

Fig. 4: First Two Spatial POD Modes extracted from CFD Data at Re = 100

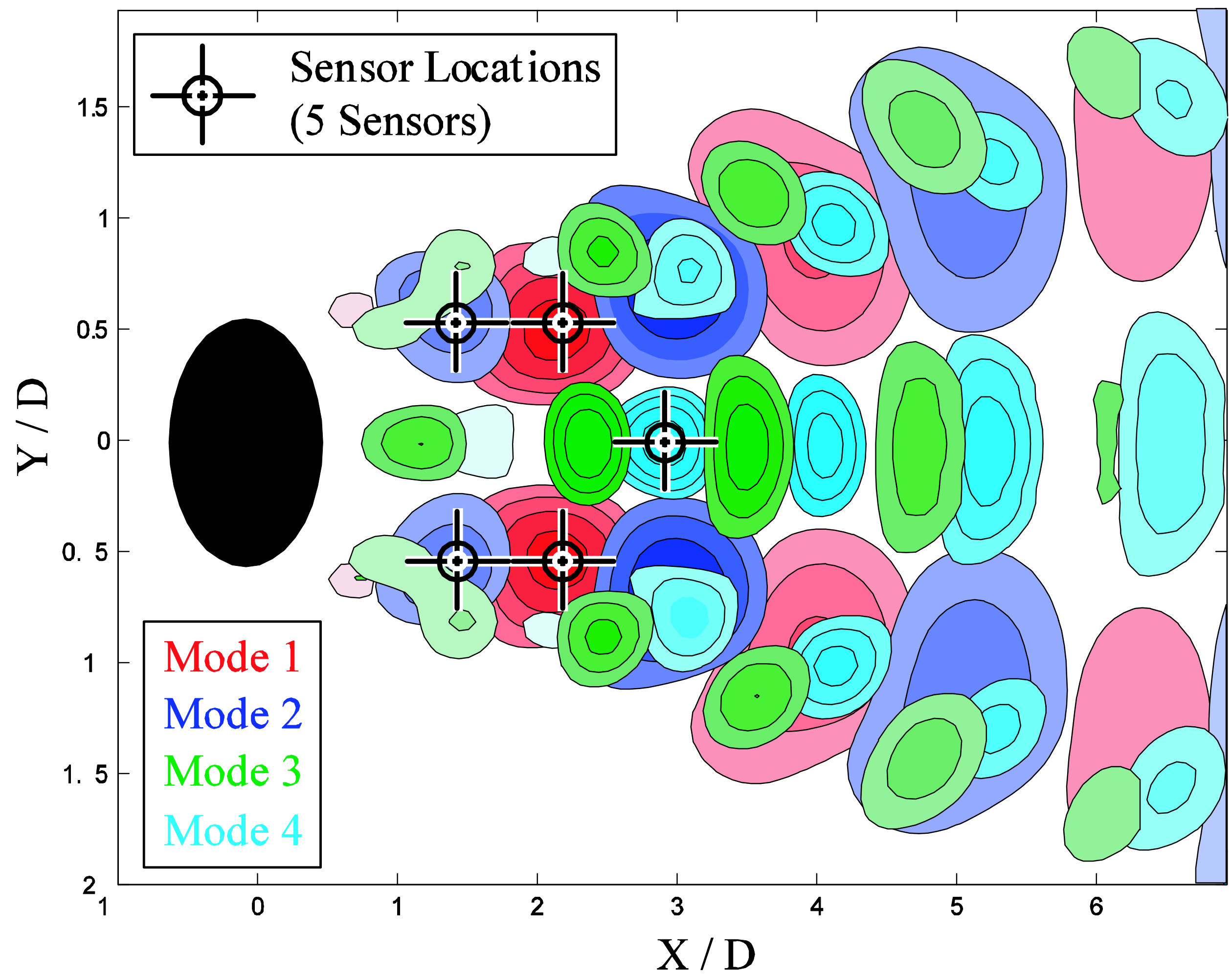

Fig. 5: Sensor location and number for estimation of first four modes at Re = 100

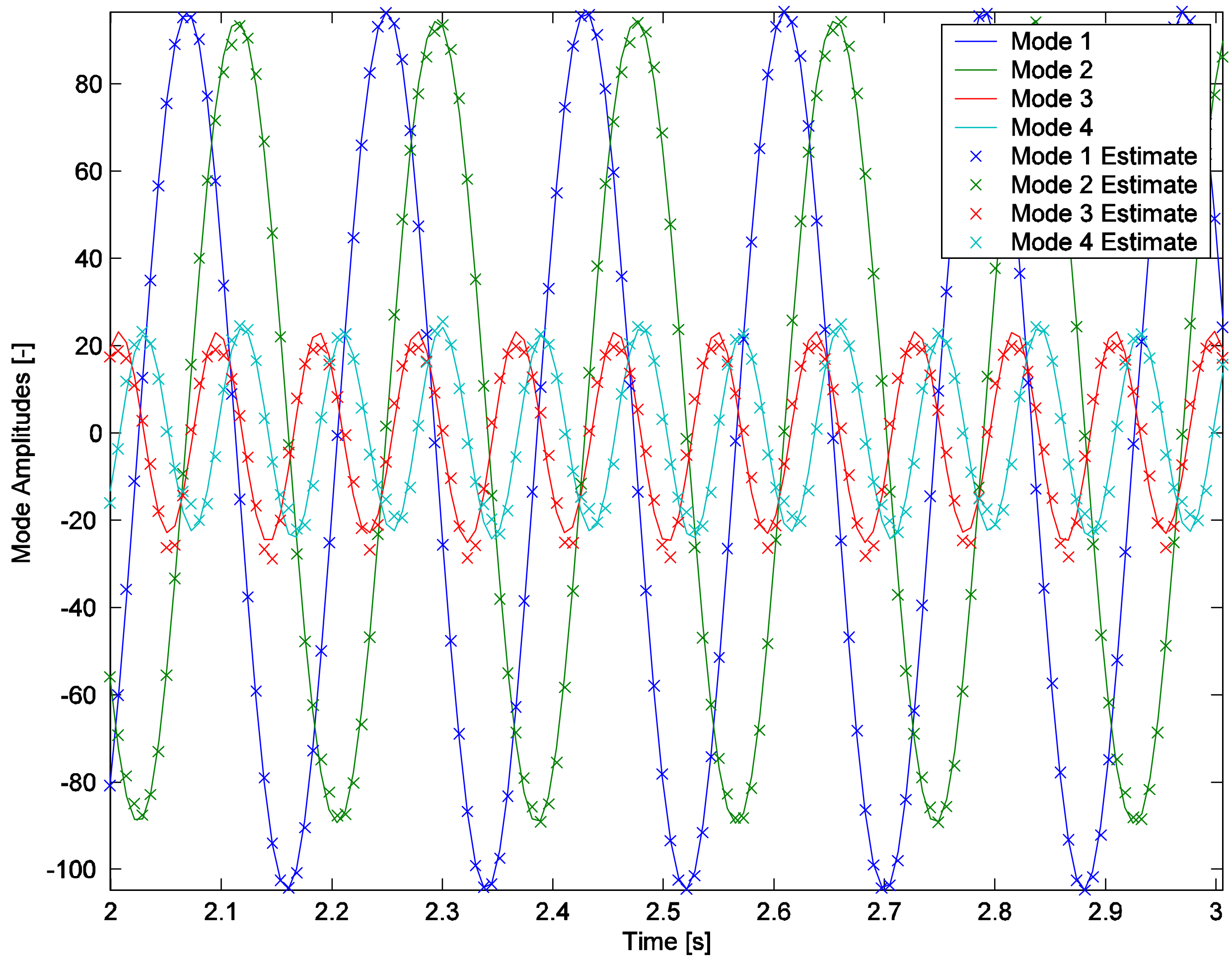

Fig. 6: Temporal Mode Amplitudes and their Estimates at Re = 100