Modeling of the Wake Behind a Circular Cylinder Undergoing Rotational Oscillations

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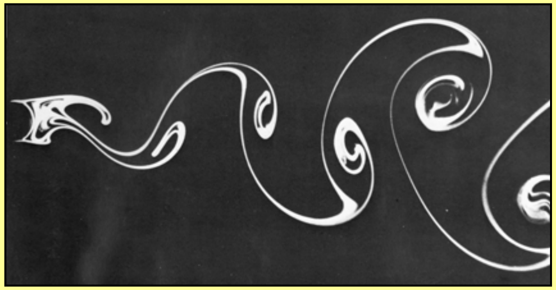
USAFA Closed-loop Flow Control Program

Part I

Flow



➔ Demonstrate *closed-loop* control of a bluff-body wake

- Circular cylinder in a cross-flow at Re_D=125
  - Cylinder can rotate about its longitudinal axis: *control input*
- Using wake velocity measurements develop a reduced-order model of the wake when the cylinder oscillates
- Use this model to understand the response of the wake to the control input
- Develop a feedback control scheme that reduces *flow-induced vibration* and *drag*

Characteristics & Control of the Cylinder Wake

Characteristics

- A region of absolute instability in the near-wake gives rise to a self-sustained oscillation
  - Manifested as the Karman vortex street
  - Low-dimensional flow
- Oscillating wake flow feeds back unsteady forces to the cylinder

  → *Flow-induced vibrations*

Feedback Control

- Single sensor feedback can be effective at locally reducing wake oscillations → at sensor location
- 3-D effects from along the span re-establish shedding away from sensor (Roussopoulos & Monkewitz, 1996)
- Multi-sensor control works better but the control window closes as Reynolds number increases (Gillies, 1998)

Program

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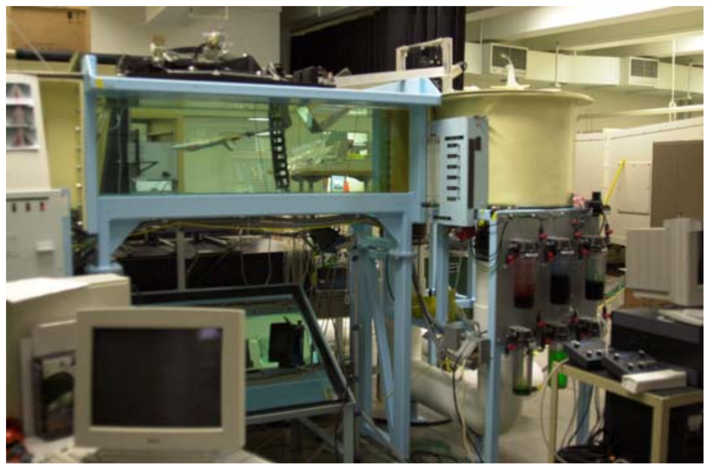
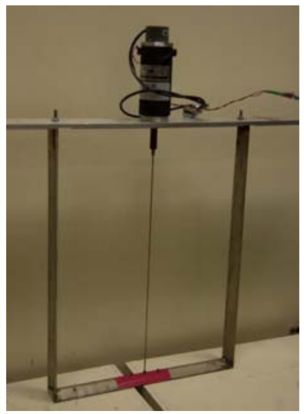
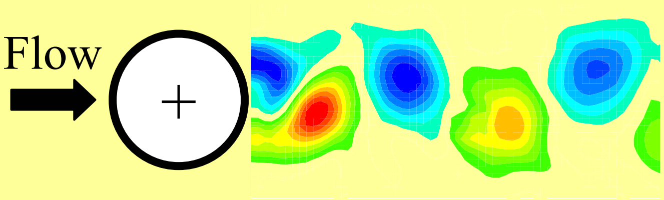
Experimental Facility & Conditions

Water channel







- Vortex shedding frequency is a fcn of Re_D

$$\left.\begin{array}{l} D = 2.38\,\text{mm} \\ U_\infty = 6.0\,\text{cm/s} \end{array}\right\} Re_D = 125 \implies S_f \equiv \frac{f\,D}{U_\infty} \approx 0.18$$

- Forcing Strouhal numbers, $S_f = 0.189$, 0.248

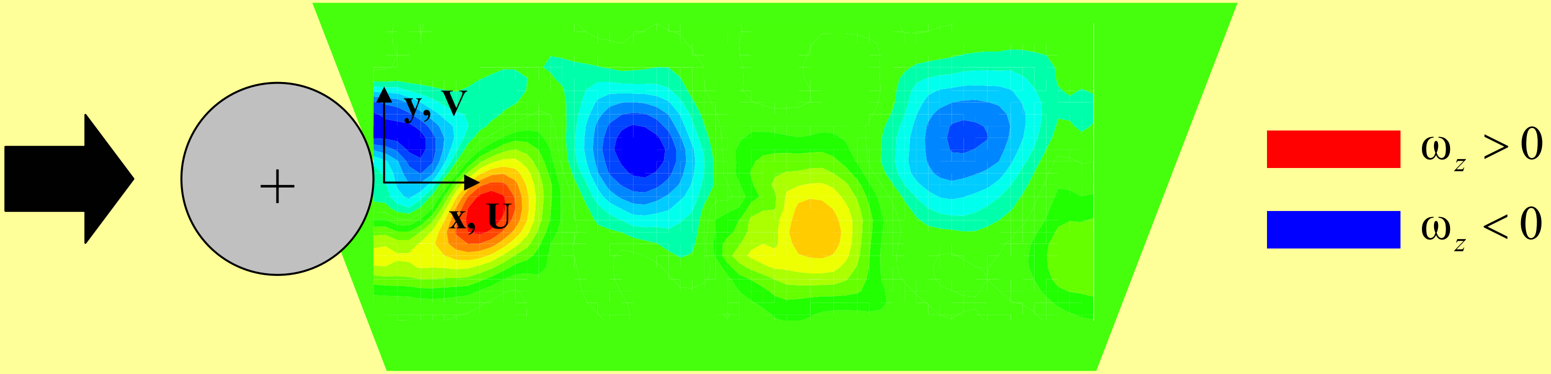
- Rotational forcing: $\Omega_1 = \dfrac{V_{\theta,max}}{U_\infty} = \dfrac{2\pi r \theta_{max} f_o}{U_\infty}$

  - Seventeen forcing conditions,

$$0.0 \le \Omega_1 \le 1.5$$

PIV Measurements

- Velocity field measurements were made with **P**article **I**mage **V**elocimetry (U, V)
  - Seed water w/ small particles (10μm diameter)
  - Illuminate particles & take two, short-exposure photographs separated in time by a small, known Δt
  - Particle displacements Δx & Δy divided by Δt give velocities (U,V)
  - Data images were acquired at 15 Hz, asynchronously & phase-averaged w/ f_o
- Contour plots show cross-span vorticity, $\omega_z = \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right)$



(View from below)

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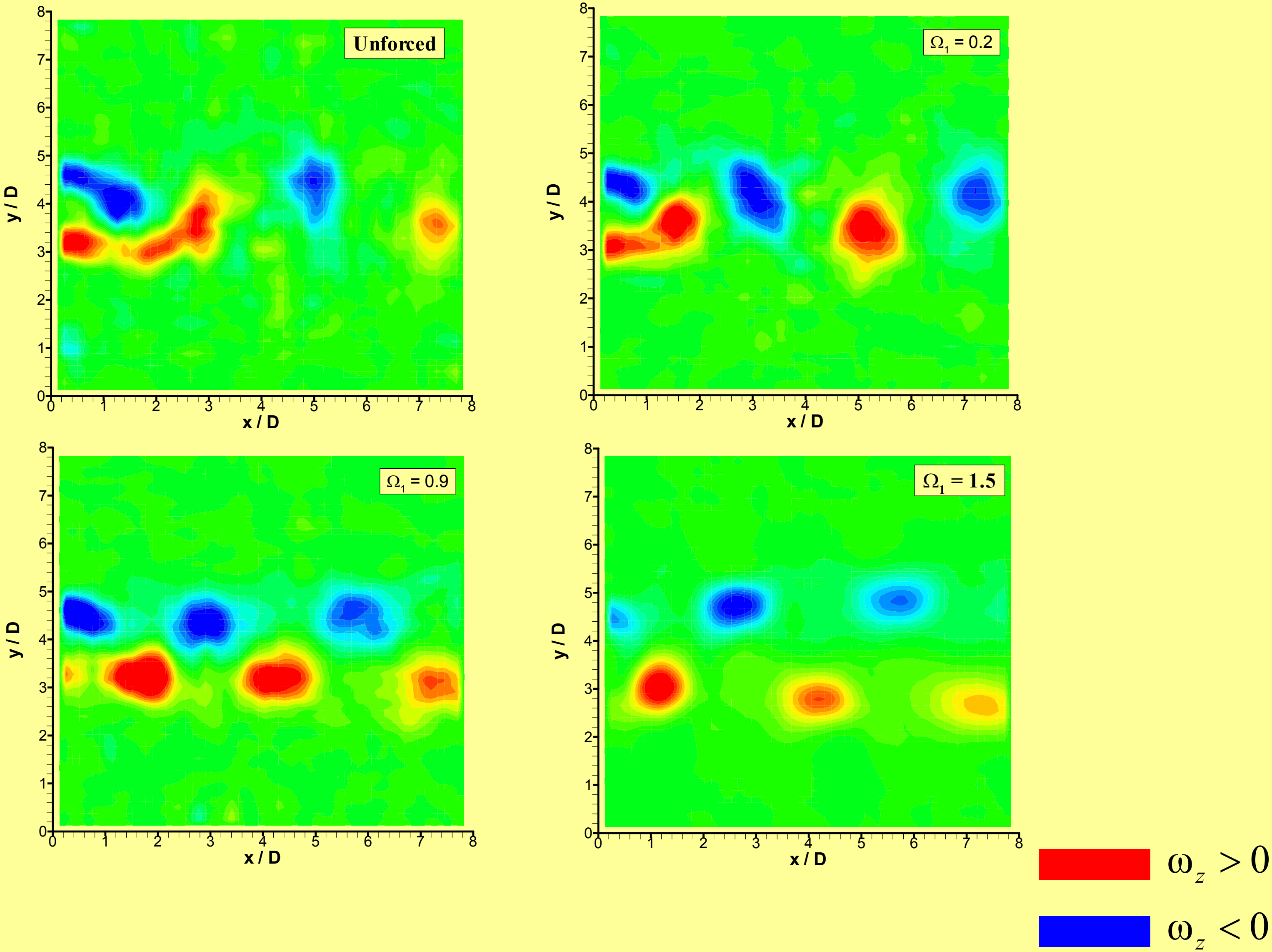
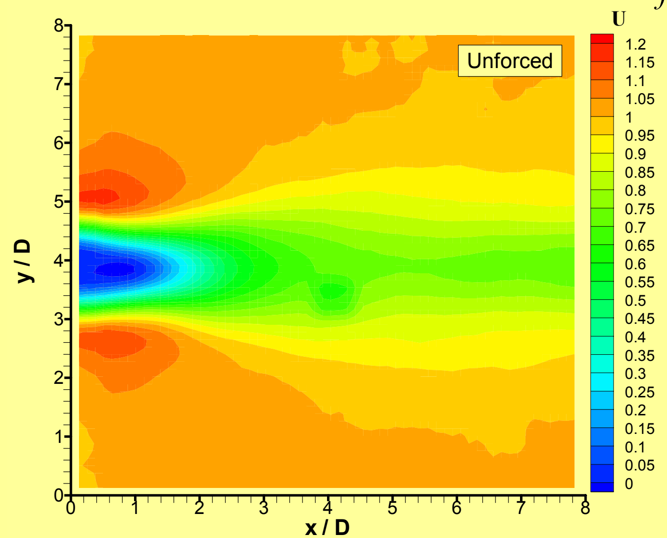
Instantaneous Vorticity Contours

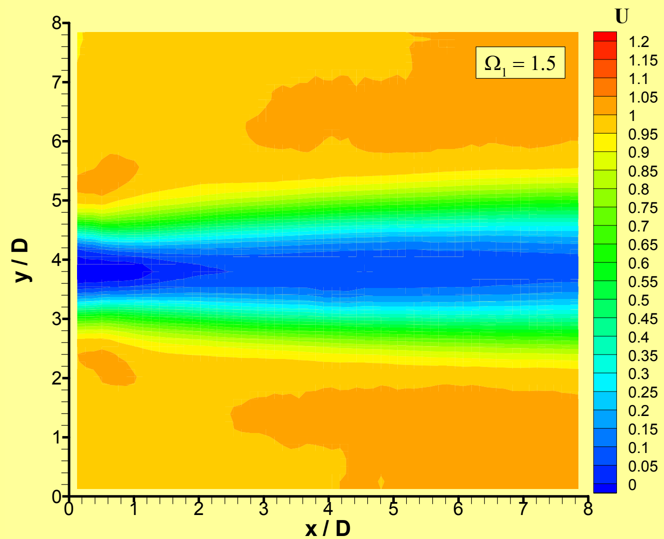
$S_f = 0.189$



Mean Wake

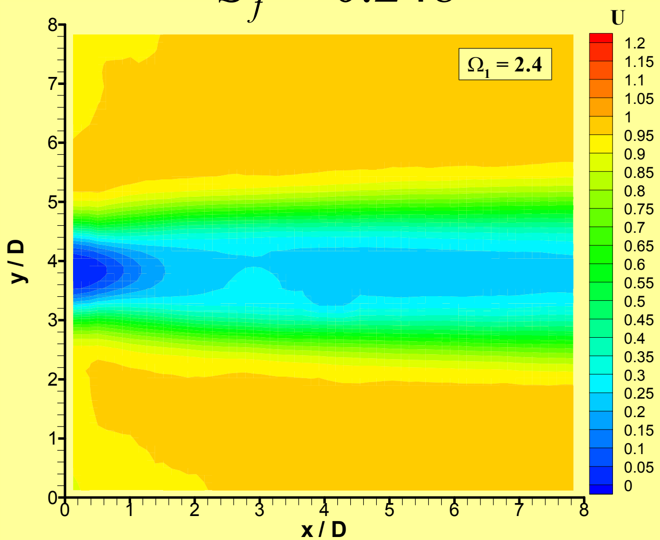
$$S_f = 0.189$$





$$S_f = 0.248$$

Contours of streamwise velocity component



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Modeling Low-dimensional Flows

(Lumley, 1967, 1981)

- Following the work of Gillies (1998) and Graham et al. (1999), develop a physics-based model of cylinder wake
- Use a Galerkin method to obtain the model

$$u_i\left(x_j, t\right) = \sum_{k=1}^{\infty} a_k(t)\, \psi_i^k\left(x_j\right)$$

  - Obtain a small set of characteristic velocity fields that possess the majority of the turbulent kinetic energy in the flow ⇒ Proper Orthogonal Decomposition
  - Project the Navier-Stokes equations onto this set to obtain a low-dimensional model of the flow
- Use this model to recreate the basic features of the wake
  - Reconstruct the large-scale vortices in the wake
  - Gain insight to the flow dynamics
  - If the model is robust, we might attempt flow control

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Proper Orthogonal Decomposition

(Holmes, Berkooz & Lumley, 1996)

- Use POD to obtain modes for Galerkin projection
- POD is an *optimal* decomposition in that it will "capture the most energy in the fewest modes"
  - Maximize the average projection of u onto Ψ

$$\max \frac{\left\langle \left| (u, \psi)^2 \right| \right\rangle}{\|\psi\|^2}$$

- From calculus of variations:

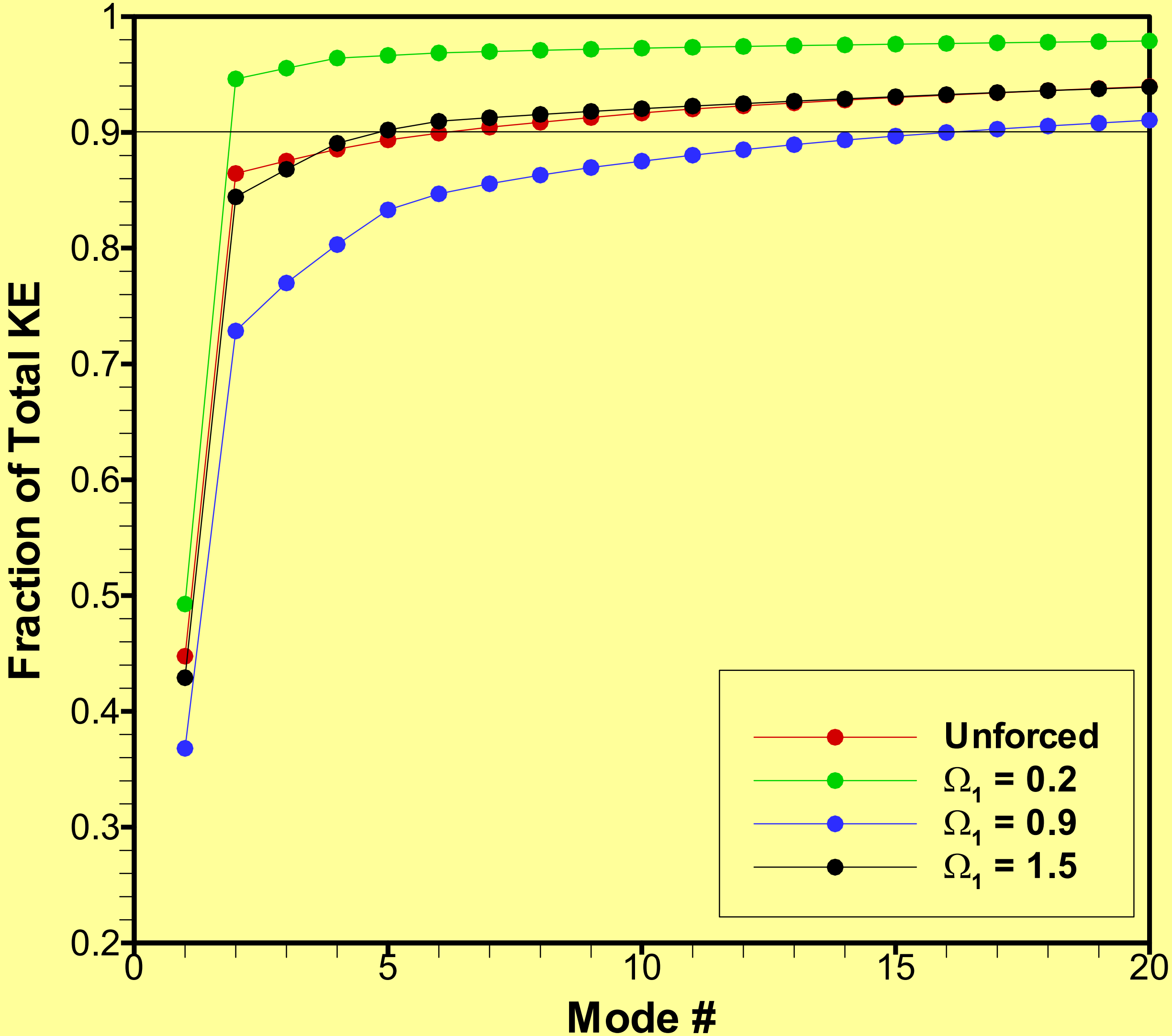
$$\int K(x, x')\,\psi(x')\,dx' = \lambda\,\psi(x)$$

where $K(x,x')$ is the two-pt velocity autocorrelation tensor, $\Psi(x)$ are the empirical eigenfunctions and λ are the empirical eigenvalues

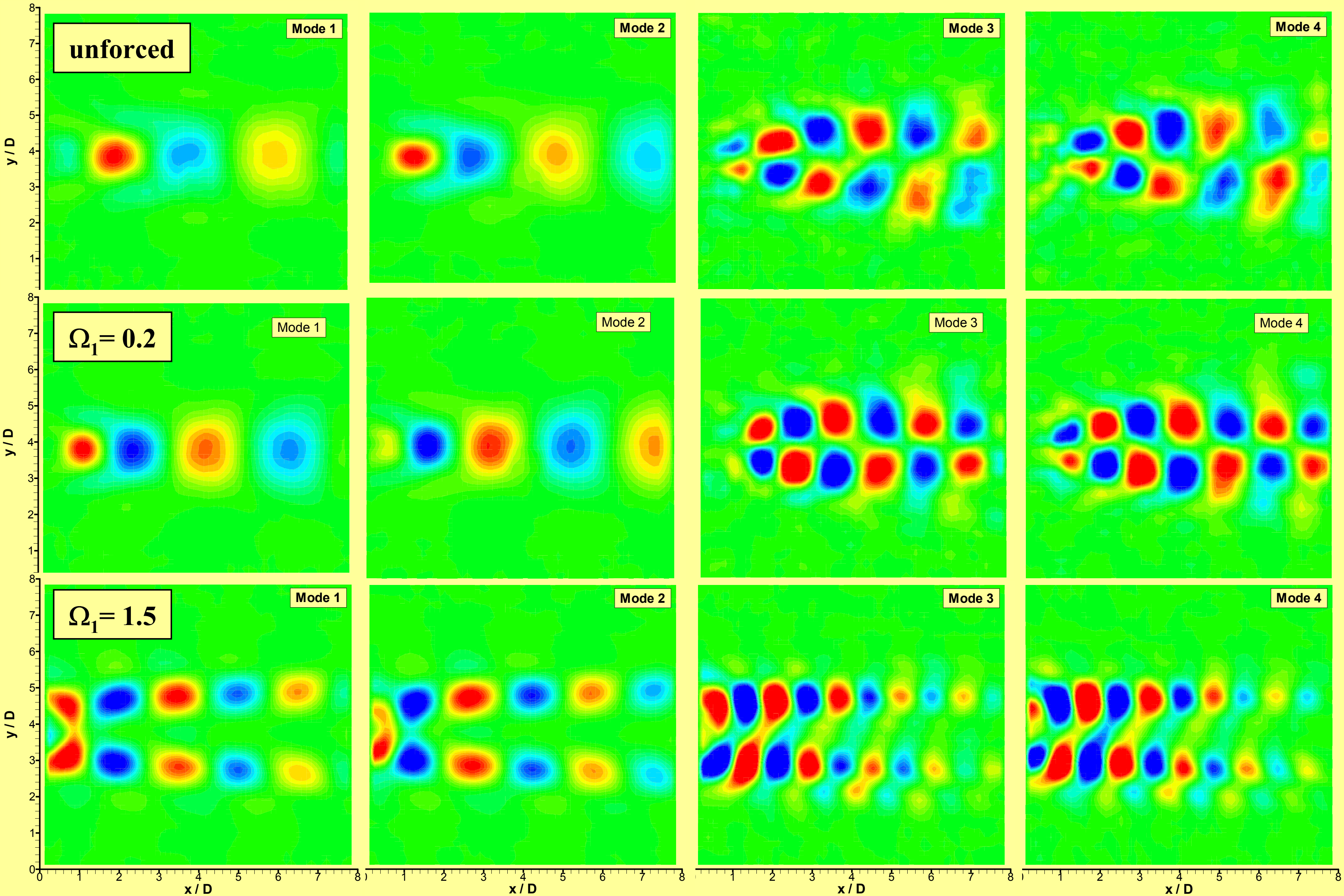
- Use "Method of Snapshots", Sirovich (1987)

Modal Energy Distribution

$S_f = 0.189$



POD Modes – Vorticity Contours



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Galerkin Projection

- Modes identify the space in which to construct a model of the flow
- Project the Navier-Stokes equations onto this space to obtain our flow-field model

$$\underbrace{u_i(x_l,t)=\sum_{k=1}^{N} a_k(t)\,\psi_i^k(x_l)}$$

$$\left(\psi_i, \frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_j}\left(U_j u_i + u_j U_i + u_i u_j - \overline{u_i u_j}\right)=\frac{1}{\mathrm{Re}_D}\frac{\partial^2 u_i}{\partial x_j^2}\right) \Rightarrow \boxed{\frac{da_k}{dt}=F(a)}$$

Navier-Stokes Eqns for fluctuation

where F is a non-linear function

- Order N of the model depends on how many modes we require to faithfully represent the large-scale turbulence

Low-Dimensional Model

- Model for a turbulent flow:

$$\frac{da_k}{dt} = -B^{kn} a_n - C^{knm}\left(a_n a_m - \overline{a_n^2}\right) + \frac{1}{\mathrm{Re}_D} D^{kn} a_n$$

where k = 1, 2, …, N

$$B^{kn} = \left(\psi_i^k, U_j \frac{\partial \psi_i^n}{\partial x_j} + \psi_j^n \frac{\partial U_i}{\partial x_j}\right)$$

→ Velocity-strain rate interactions

$$C^{knm} = \left(\psi_i^k, \psi_j^n \frac{\partial \psi_i^m}{\partial x_j}\right)$$

→ Fluctuating velocity interactions

$$D^{kn} = \left(\psi_i^k, \frac{\partial^2 \psi_i^k}{\partial x_j^2}\right)$$

→ Viscous term

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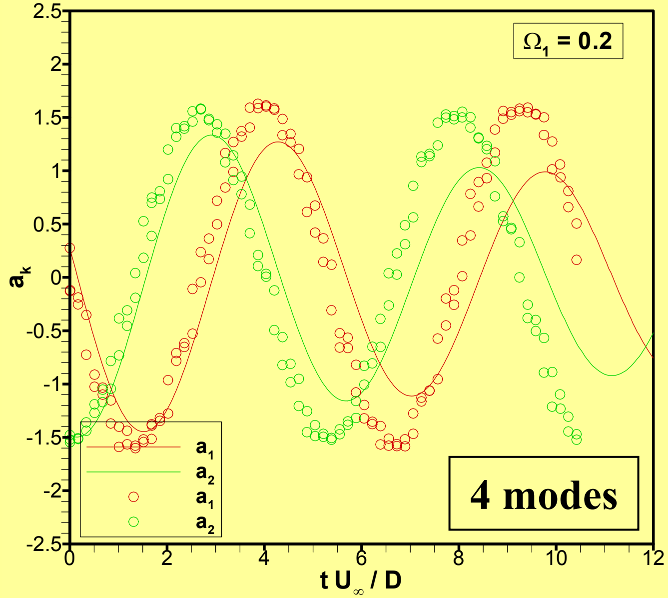
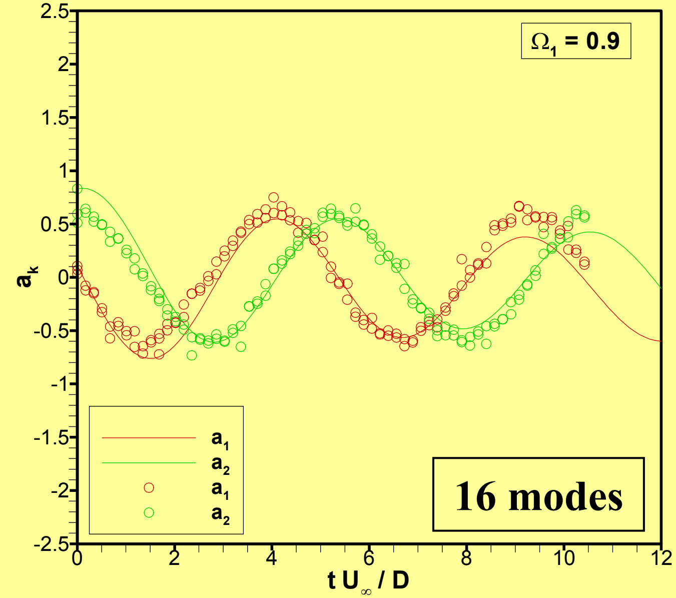
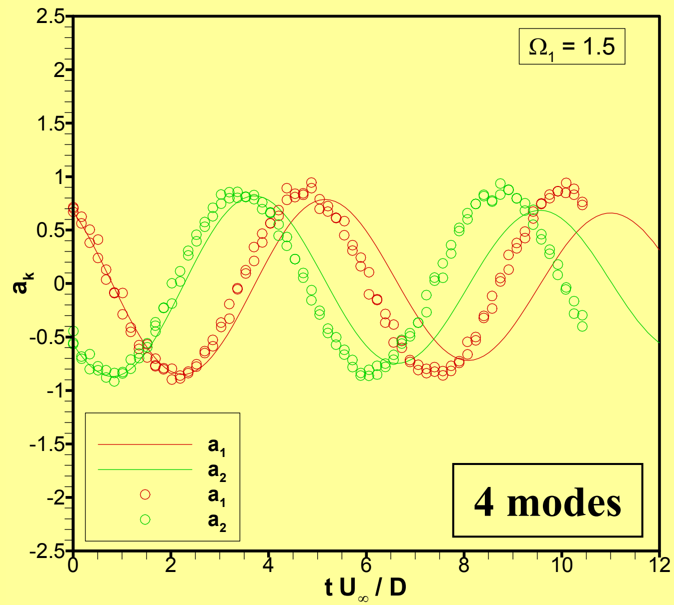
Evaluation of Model Predictions

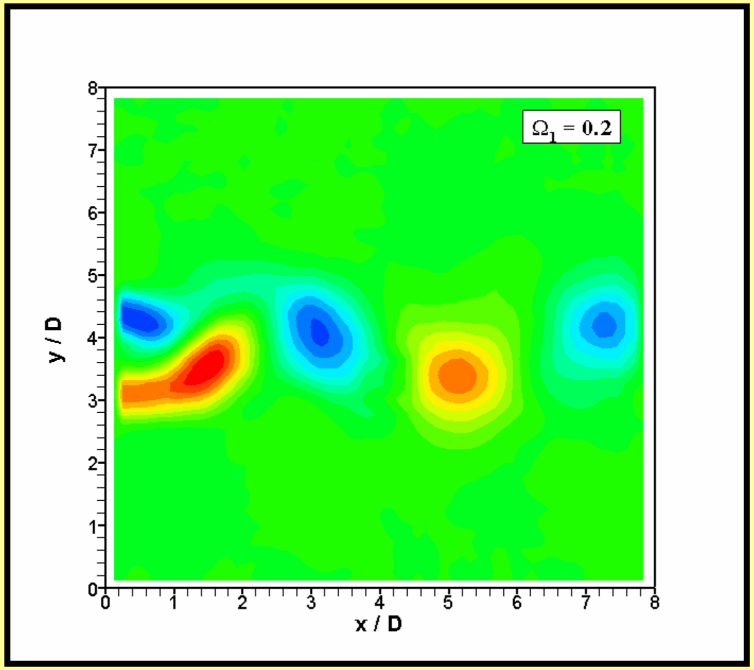


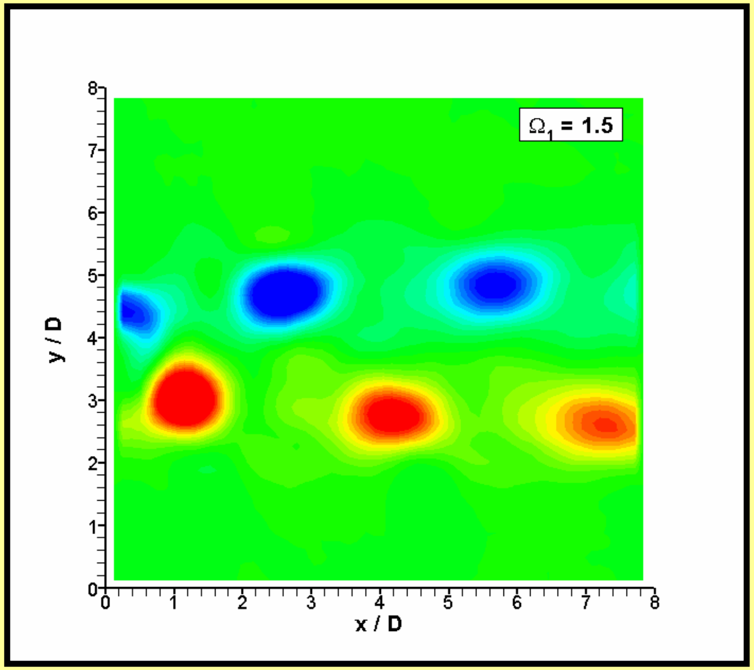


- Symbols are projections of experimental data
- Lines are model predictions



Reconstruction of the Wake





$$\mathrm{U}_i(x_l,t)=\left\langle \mathrm{U}_i(x_l)\right\rangle + u_i(x_l,t)$$

$$u_i(x_l,t)=a_1(t)\,\psi_i^1(x_l)+a_2(t)\,\psi_i^2(x_l)+a_3(t)\,\psi_i^3(x_l)+a_4(t)\,\psi_i^4(x_l)$$

$\omega_z > 0$

$\omega_z < 0$

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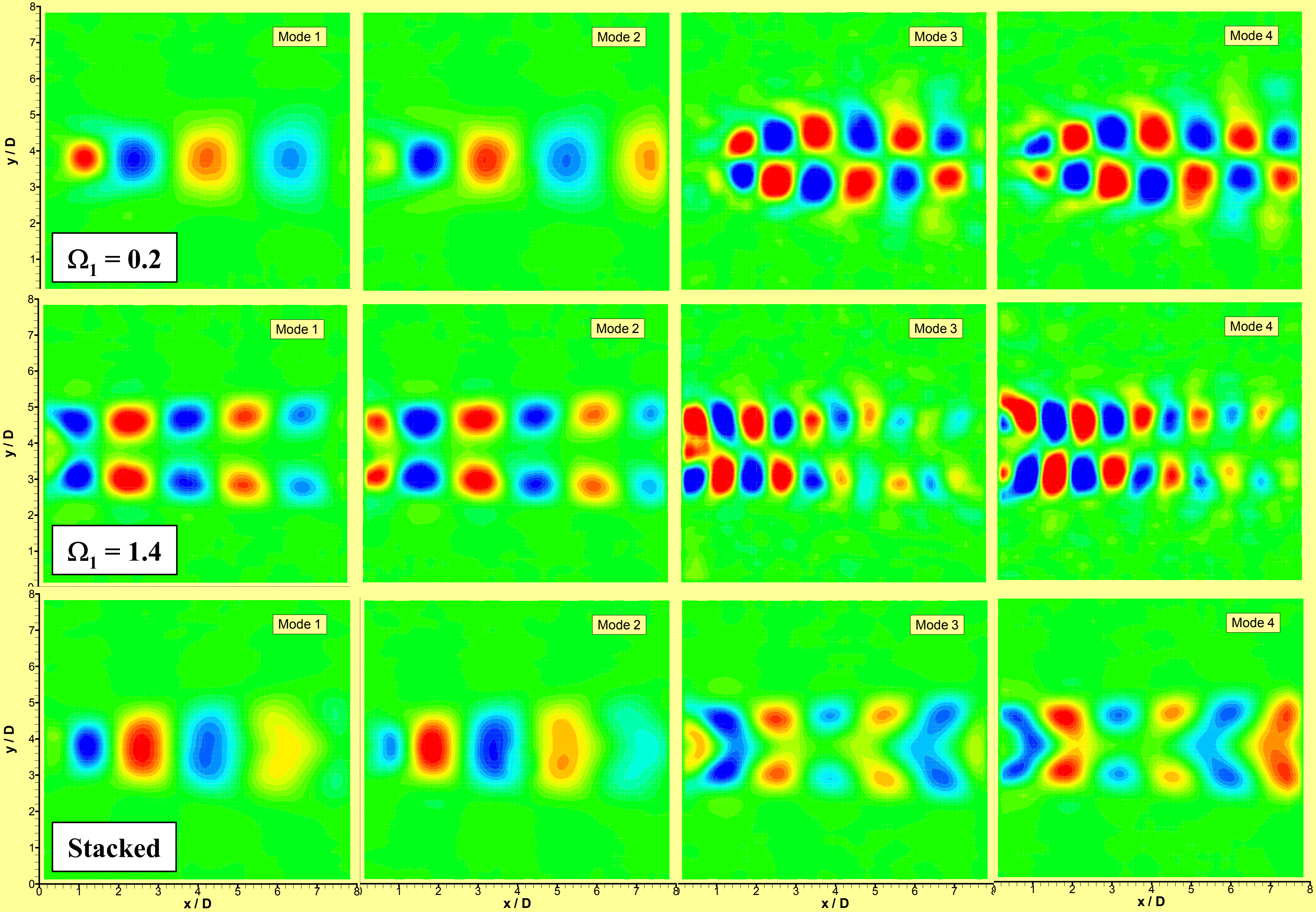
Introduction to Data Stacking

- POD analysis for $\Omega_1 = 0.2$ and 1.5 reveal distinct differences in the mode sets
  - A set of modes for one forcing condition may not well-represent the flow dynamics for a different forcing condition
  - How can we obtain a set of modes that spans a range of forcing conditions? *Use a stack of data sets from different Ω_1*
- A much larger set of data is used in the determination of the POD modes
  - Improves the velocity correlation statistics
  - Yields a basis that presumably is applicable to a wider variety of flow conditions
- Use data sets for $\Omega_1 = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2$ and 1.4 to obtain the POD modes, then model one of the flows

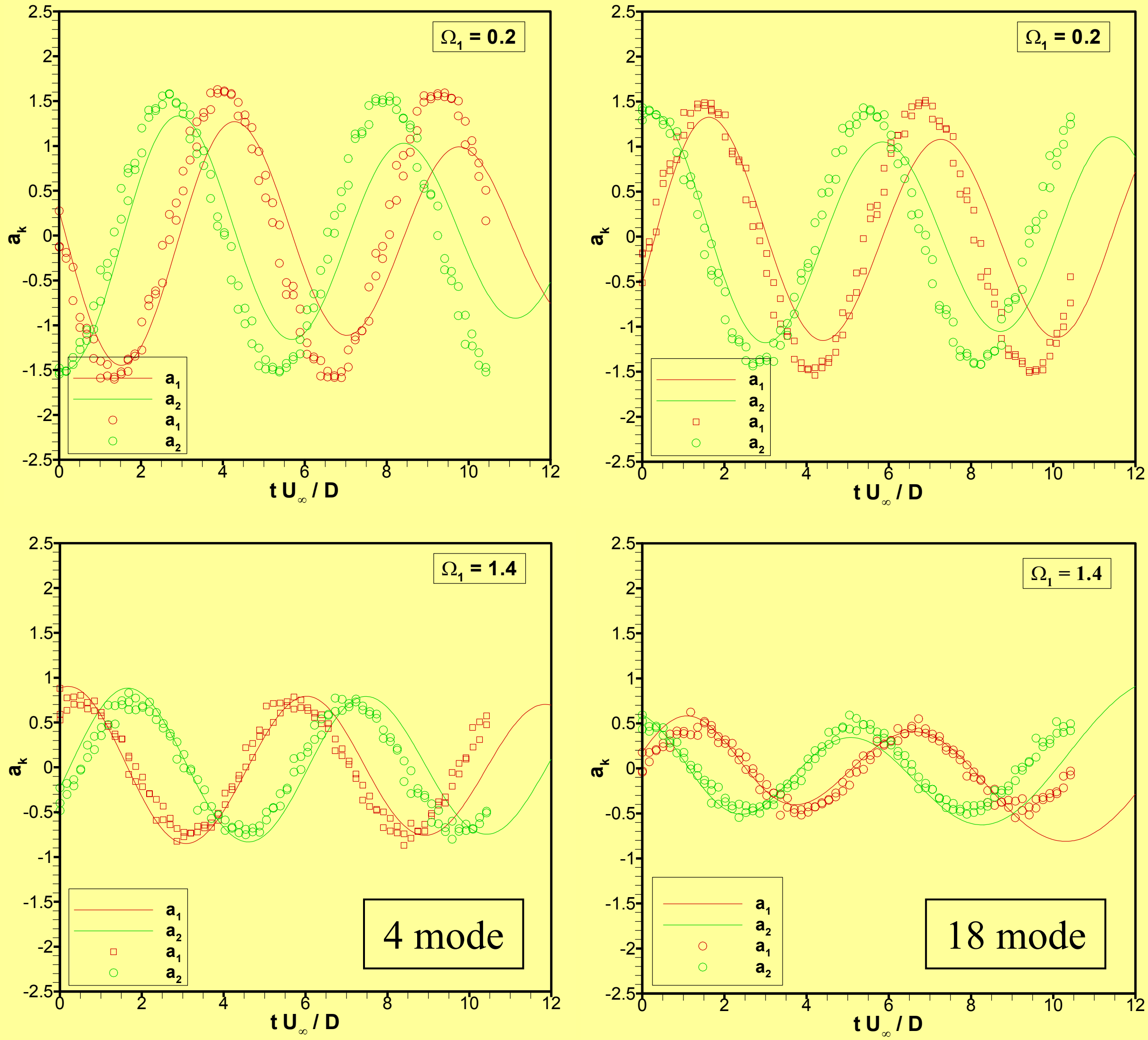
Comparison of Stacked POD Modes



Comparison of Low-D Model Predictions

4 mode models

4 & 18 mode models

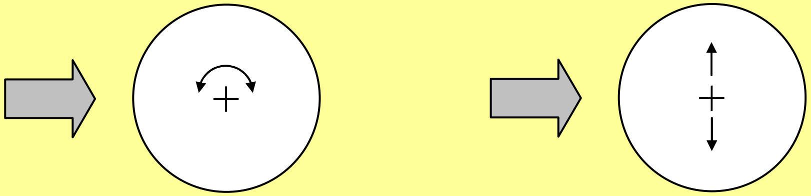


Single forcing modes

Stacked modes

Continuing Work

- Model development
  - Implement a low-dimensional model with control fcn
  - Incorporate 3-D effects into the model
  - Better understand the wake response to oscillations at the cylinder
- Continuing experimental work: rotational/transverse oscillations



UW experiments   USAFA experiments

- Integrate CFD results into the analysis & modeling
- Demonstrate closed-loop control of the wake

Closed-loop Flow Control Program

CONTROL PROBLEM

Vortex Suppression in Cylinder Wake

MODELING THE CONTROLLED FLOW

- **Truth Model – CFD (Cobalt), USAFA**
- **Evaluation Model – Low-dimensional, POD**

ESTIMATOR

Real-time mapping of PIV measurements onto POD states

CONTROLLER

Real-time mapping of POD states to actuator commands

CONTROL STRATEGY VERIFICATION

- **Truth Model – CFD (Cobalt)**
- **Water Tunnel Test & Evaluation**