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Feedback Control of a Cylinder Wake Low-Dimensional Model

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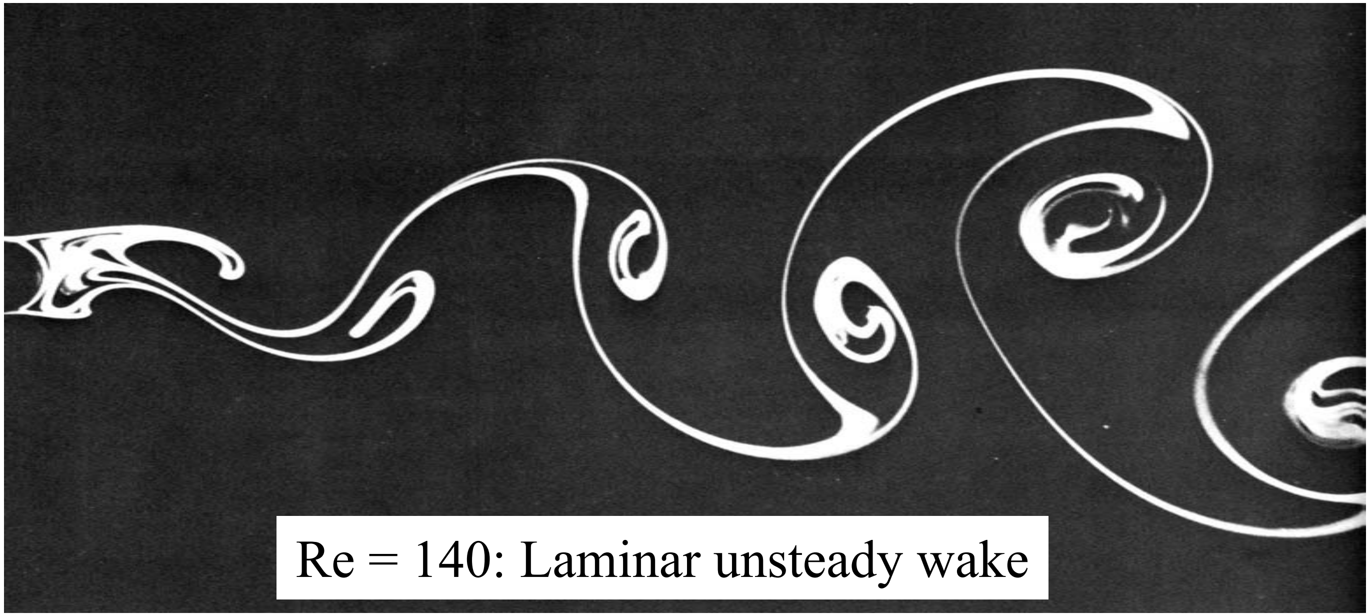
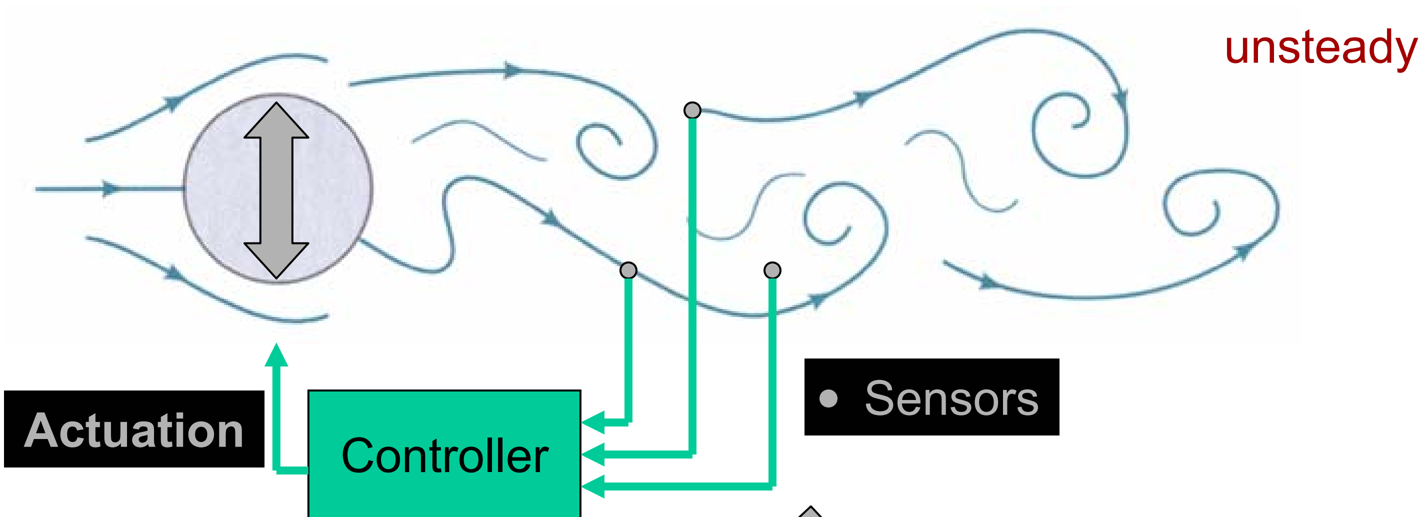
INTRODUCTION

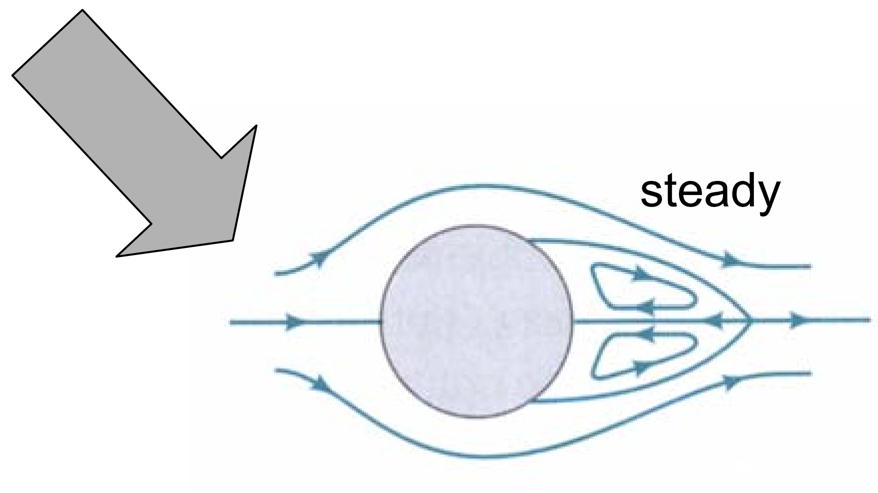


Re = 140: Laminar unsteady wake

- In a two-dimensional cylinder wake, self-excited oscillations in the form of periodic shedding of vortices are observed above a critical Reynolds number of around 50.
- These flow-induced non-linear oscillations lead to some undesirable effects associated with unsteady pressures such as fluid-structure interactions.
- The only way of suppressing the self-excited flow oscillations is by the incorporation of active closed-loop flow control.

RESEARCH OBJECTIVE

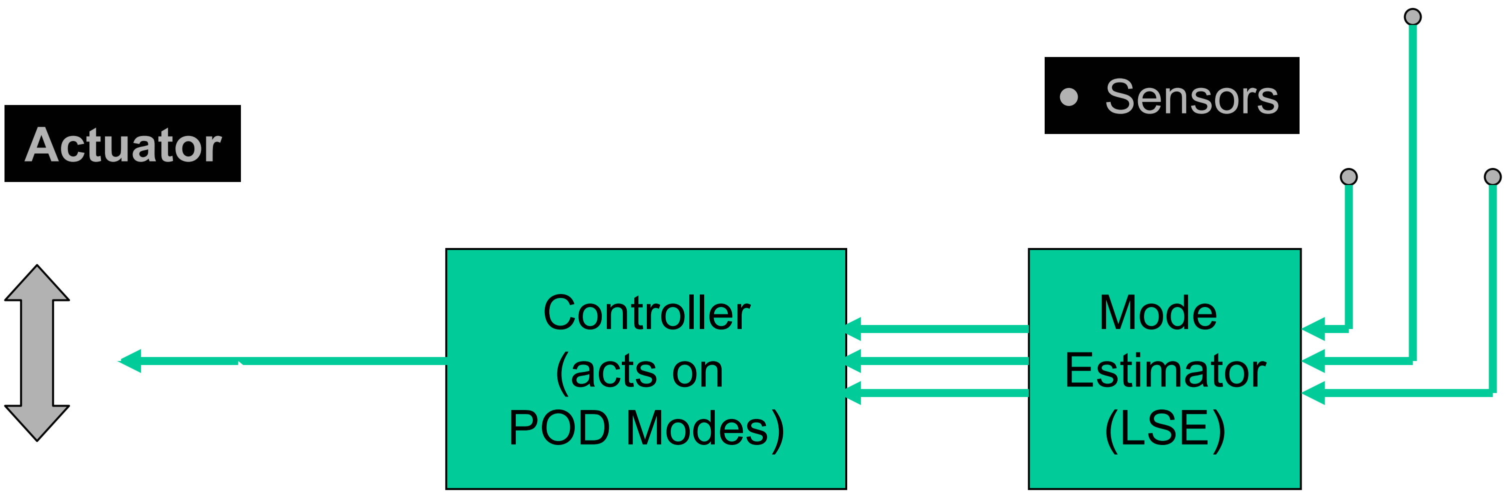




Develop a closed-loop control strategy to suppress the Karman vortex street of a cylinder at Reynolds numbers of 100-120.

CONTROL APPROACH

- **Low-dimensional model identification of the closed-loop system based on proper orthogonal decomposition (POD).**



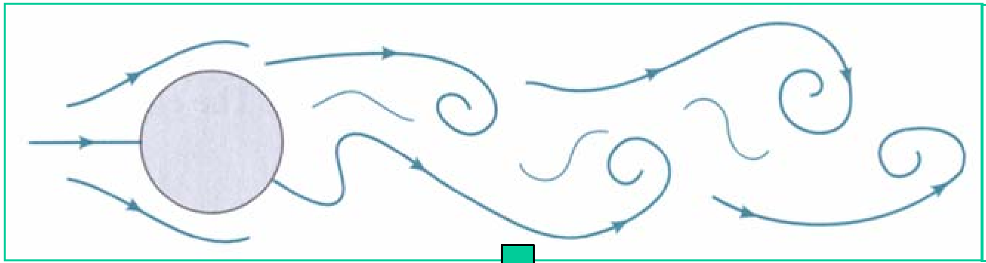
Actuator Command (Displacement, Velocity)

Flow Modes (POD Amplitudes)

Sensor Information (Velocity, Pressure)

LOW DIMENSIONAL MODELING

Re = 100



K Snapshots of Flow Field
U(x,y,t)
V(x,y,t)

4 Mode POD Model based on DNS CFD simulations

K Temporal Mode Amplitudes
$A_1(t)$
$A_2(t)$
…
$A_K(t)$

The control input was added to the POD model as an external source that emulates the translation actuation of the cylinder

N Spatial Modes
M_1_U(x,y)
M_1_V(x,y)

M_2_U(x,y)
....
M_K_V(x,y)

LOW-DIMENSIONAL POD MODEL

- Based on the temporal coefficients of the POD model, the following system of partial differential equations was developed using a least squares fit:

$$\dot{A}_k = g_k(A_1, A_2, \ldots A_M) + b_k f_a \qquad \text{(for } k = 1,2,\ldots.M)$$

- The function g_K is chosen to be cubic:

$$g_k = c_0^k + c_{1i}^k A_i + c_{2ij}^k A_i A_j + c_{3ijl}^k A_i A_j A_l \qquad \text{(for } i,j,k,l = 1,2,\ldots M)$$

A_k - time-dependent coefficient of the kth mode

M - total number of modes in the low-dimensional model

g_k - nonlinear function of the time-dependent coefficients

f_a - control input to the cylinder

b_k - represents the coefficients associated with the control input

CONDITIONS FOR CONTROLLABILITY

- For nonlinear system $\dot{A}_K = g_K(A_1, A_2, \ldots A_M) + b_K f_a$
- The simplest approach to study controllability is to consider its linearization

$$\dot{A}_K = J \cdot A_j + b_K \cdot f_a \qquad \text{Let } B = [b_1, b_2, \ldots b_M]$$

- Definition: *The pair (J, B) is state controllable if and only if there exists a control f_a that will transfer any initial state $A_K(t = 0)$to the desired equilibrium point in finite time.*
- We will demand that the following algebraic condition for controllability:

$$\operatorname{rank}(B \vdots JB \vdots J^2B \vdots \cdots \vdots J^{n-1}B) = n$$

CONTROL APPROACH

- It is desirable to develop estimator and controller strategies that are simple yet effective.
- **Let** the control law be based on the estimate of **only one mode** as follows:

$$f_a = -K_P \cdot A_1^{est}$$

- where K_P is the proportional gain of the P controller and A_1^{est} is the estimate of the time-dependent coefficient of Mode 1, A_1 extracted from sensors placed in the wake .
- An accurate linear stochastic estimator for the first four POD modes of a circular cylinder wake was developed based on five sensors in the flow field.
- Based on this experience, in this effort estimation errors are neglected and it is assumed that $A_1 \sim A_1^{est}$.
- **Questions raised:**
  - **Is stability of all POD states assured for such a controller?**
  - **How are the gains of the controller determined?**
  - **Is such an approach effective?**

LINEARIZATION OF THE POD MODEL

- The function g_k is expanded locally as a Taylor series about the desired equilibrium point: $A_k = 0$
- Inserting the proposed proportional control law into the linearization of A_k yields:

$$\dot{A}_k = J_C \cdot A_j$$

- J_C is the "closed-loop" Jacobian and a linear stability analysis based on J_C will provide an insight into the behavior of the closed-loop system.

$$J_C = \begin{bmatrix} \frac{\partial g_1(0)}{\partial A_1} - b_1 . K_P & \cdots & \frac{\partial g_1(0)}{\partial A_M} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_M(0)}{\partial A_1} - b_M . K_P & \cdots & \frac{\partial g_M(0)}{\partial A_M} \end{bmatrix}$$

STABILIZATION OF THE POD MODEL

- The conditions for asymptotic stability for linearized models about their equilibrium point, as follows: ***if the Jacobian, J_C, has n eigenvalues, each of which has a strictly negative real part, then the equilibrium point is asymptotically stable.***
- The Hartman-Grobman theorem states that the local phase portrait near a hyperbolic (all the eigenvalues of the linearization lie off the imaginary axis) fixed point is “topologically equivalent” to the phase portrait of the linearization.
- A linearized system that is hyperbolic is equivalent in terms of stability and bifurcations, chaos and attractors, equilibria and limit cycles to the nonlinear POD model.
- From a practical point of view, it is the aim of the control design to find an appropriate gain, K_P, which will render ***all*** the eigenvalues of J_C to have a ***negative real part.*** In addition, the eigenvalues need to lie off the imaginary axis by an adequate margin so that the system is hyperbolic.

CONTROLLABILITY TEST
Based on 4 Mode POD Model

Controllability condition: $\mathrm{rank}(B \vdots JB \vdots J^2B \vdots \cdots \vdots J^{n-1}B) = n$

where n = 4 (number of modes in the prototype wake model)

The matrices J and B are extracted from the above model

$$J = \begin{bmatrix} 0.0065 & 0.1447 & 0.0518 & -0.0258 \\ -0.1293 & 0.0065 & 0.0060 & 0.0364 \\ -0.0008 & -0.0000 & -0.0347 & -0.3156 \\ 0.0003 & 0.0007 & 0.2433 & -0.0292 \end{bmatrix} \qquad B = \begin{bmatrix} -9.998e-03 \\ -9.866e-03 \\ -6.0577e-03 \\ -6.632e-03 \end{bmatrix}$$

We observe that $\mathrm{rank}(B \vdots JB \vdots J^2B \vdots \cdots \vdots J^{n-1}B) = 4$

Therefore, the pair (J, B) is state controllable

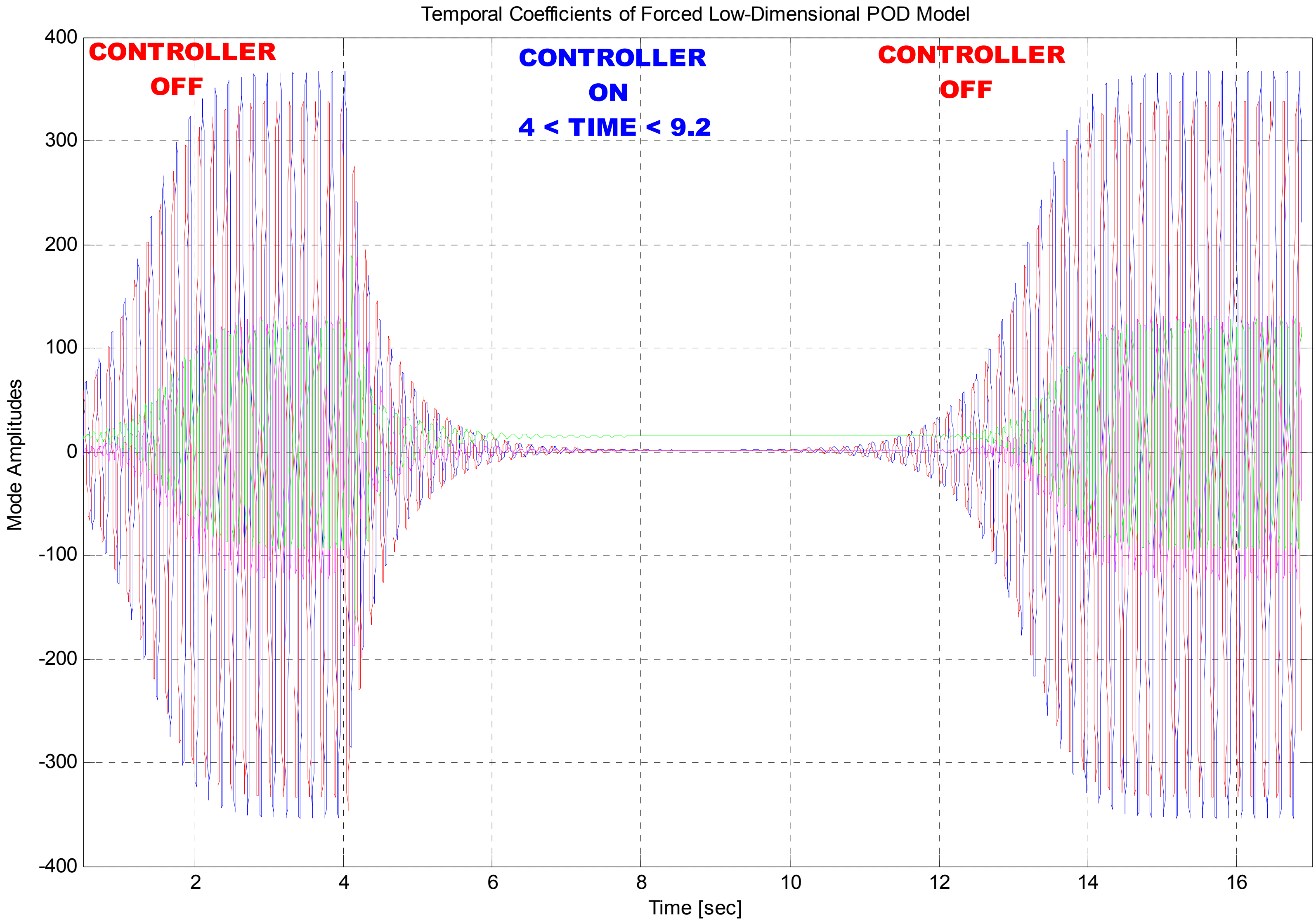
CLOSED-LOOP SIMULATIONS

- MATLAB simulations were run to find an appropriate gain, K_P, that will render ***all*** the eigenvalues of the closed-loop Jacobian, J_C, to have a ***negative real part.***

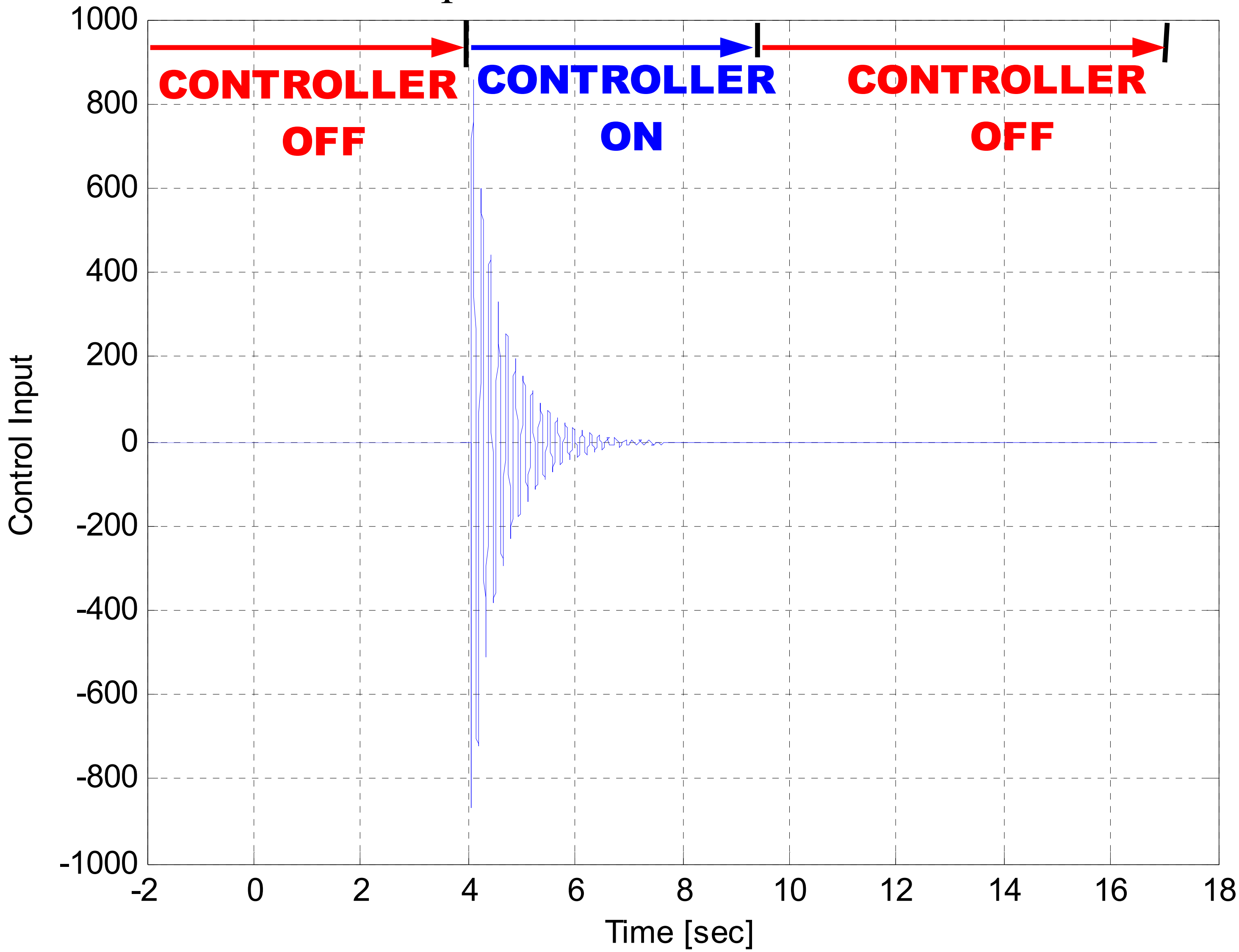
Kp	Eigenvalues 1 & 2	Eigenvalues 3 & 4
-1.2	0.0115 ± 0.1300i	-0.0310 ± 0.2768i
-0.8	0.0098 ± 0.1323i	-0.0313 ± 0.2769i
-0.4	0.0081 ± 0.1345i	-0.0316 ± 0.2770i
0.0	0.0065 ± 0.1367i	-0.0320 ± 0.2772i
0.4	0.0048 ± 0.1388i	-0.0324 ± 0.2773i
0.8	0.0032 ± 0.1409i	-0.0327 ± 0.2774i
1.2	0.0016 ± 0.1429i	-0.0331 ± 0.2775i
1.6	-0.0000 ± 0.1449i	-0.0335 ± 0.2776i
2.0	-0.0016 ± 0.1468i	-0.0339 ± 0.2777i
3.0	-0.0055 ± 0.1514i	-0.0350 ± 0.2779i

Therefore, for $K_P > 1.6$, the closed loop system is stable

CLOSED-LOOP SIMULATIONS



Control Input Based on Estimate of Mode 1



CONCLUSIONS

- For the low-dimensional POD model of a cylinder wake instability, conditions for controllability and stabilization were developed.
- A simple approach, based on the proportional feedback of the estimate of the first POD mode, has been introduced for the control of the temporal growth of the forced prototype modes.
- The above control approach was applied to a POD model developed using data obtained from a Direct Navier Stokes CFD simulation of a cylinder wake at Re = 100.
- The control approach, simulated in MATLAB using the 4 mode cylinder wake model, shows desired closed-loop behavior.
- While the controller uses only the estimated amplitude of the first mode, all four modes are stabilized. This suggests that the higher order modes are caused by a secondary instability. Thus they are suppressed once the primary stability is controlled.
- The control approach stabilizes the wake fully. Once stabilized, the required actuation amplitude drops to 1/1000th of the initial value. This means that very little energy is required to maintain a stable wake.