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Observability Conditions for POD Modes in a Circular Cylinder Wake

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Outline

- Introduction and Background
  - Cylinder Wake Feedback Control
  - Controller Design and Approach
- Numerical Methods
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  - State Space Model
  - Linear Stochastic Estimation
  - Derivation of Observability Conditions
- Application of Methods to Experimental Data
- Conclusions

Introduction & Motivation

- A major issue of concern in the closed-loop control of the cylinder wake is the number and placement of the sensors.

- Furthermore, it is imperative to address the necessary conditions for "observability".

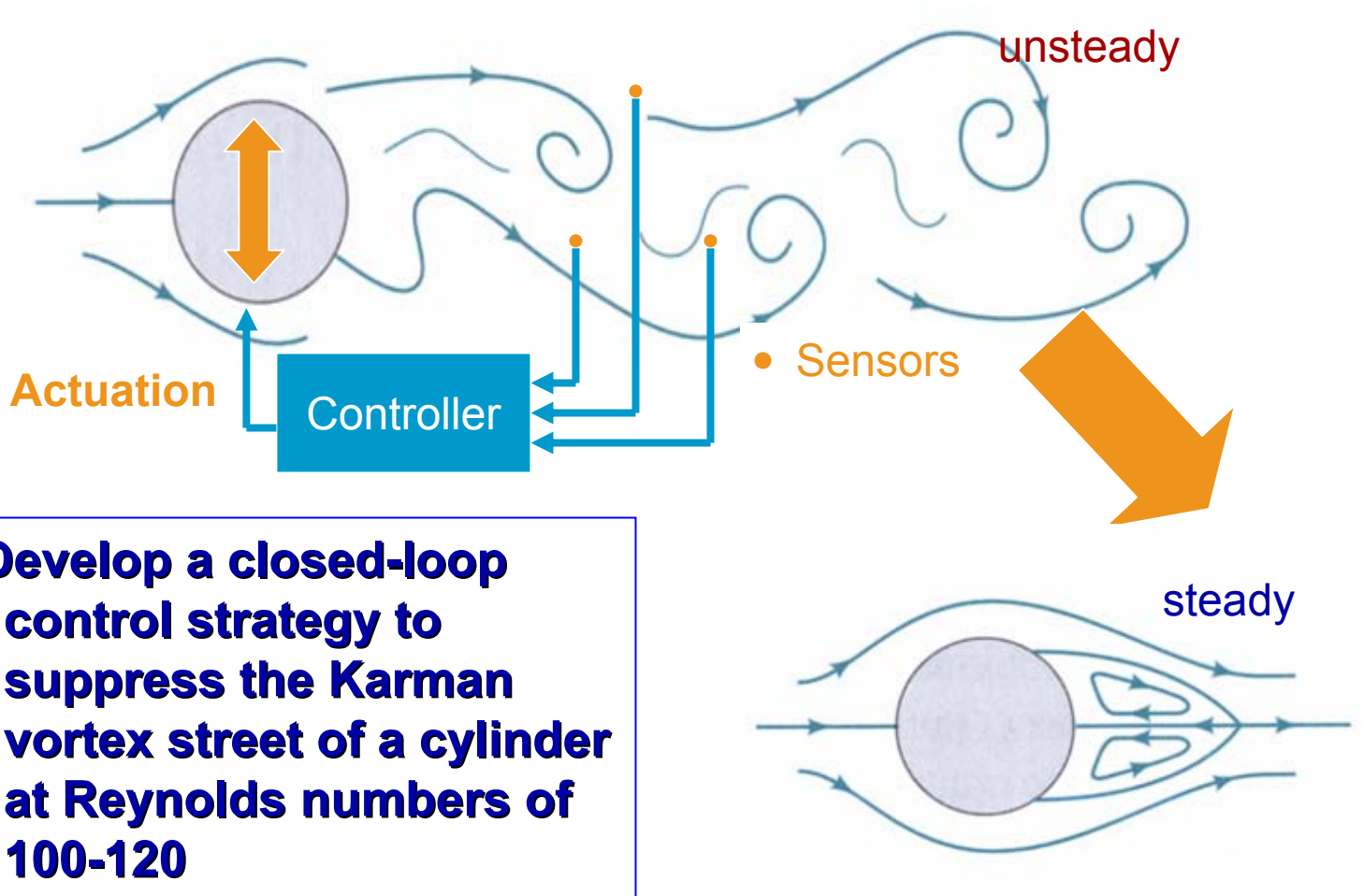
- For the case when the control strategy is based on a low-dimensional POD model, it is imperative to "observe" the behavior of the dominant modes (as required by the controller) and provide the respective estimate.

- The questions addressed in this research effort are:

  "Is there a way to be assured that a certain set of sensors will provide the necessary observation of the dominant modes?"

  "What are the necessary conditions for observability?"

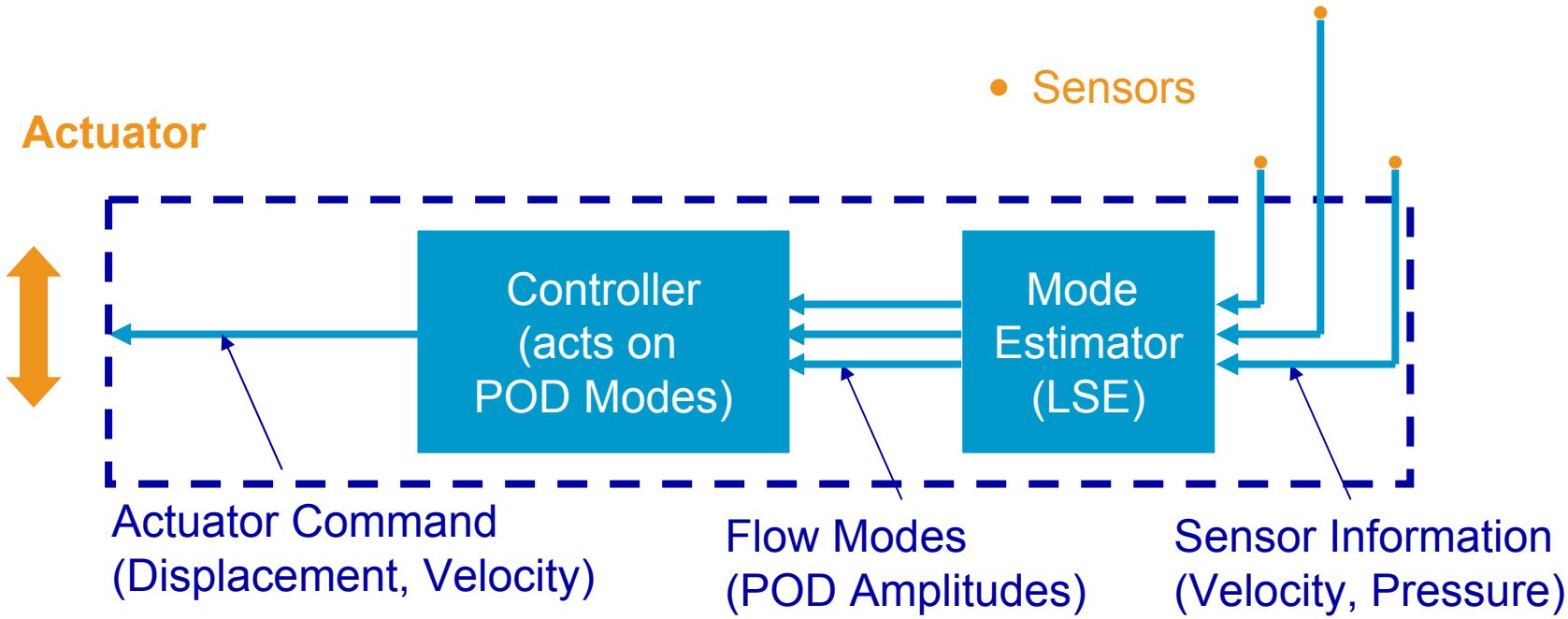
Cylinder Wake Feedback Control



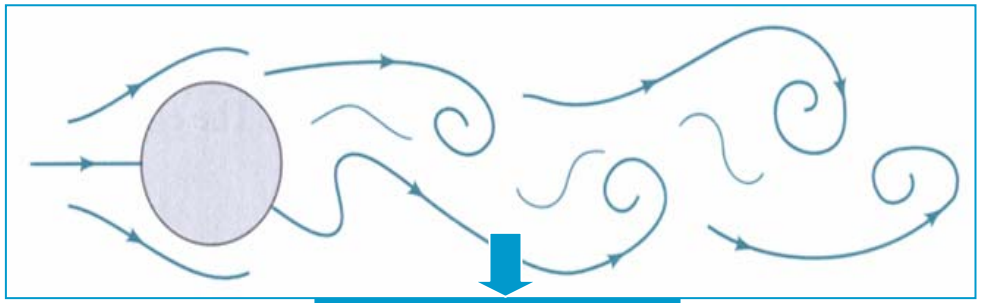
Develop a closed-loop control strategy to suppress the Karman vortex street of a cylinder at Reynolds numbers of 100-120

Sketches from: Munson, Young, Okiishi. *Fundamentals of Fluid Mechanics*. p 601.

Control Strategy



POD



N Snapshots of Flow Field
U(x,y,t)
V(x,y,t)

POD (Proper Orthogonal Decomposition)

K Temporal Mode Amplitudes
$A_1(t)$
$A_2(t)$
…
$A_N(t)$

N Spatial Modes
$M_1_U(x,y)$
$M_1_V(x,y)$

$M_2_U(x,y)$
....
$M_N_V(x,y)$

Low-Dimensional Model of the Cylinder Wake

- The dynamics of the temporal mode amplitudes of the POD Model for the turbulent flow:

$$\frac{da_k}{dt} = -B^{kn} a_n - C^{knm}\left(a_n a_m - \overline{a_n^2}\right) + \frac{1}{Re_D} D^{kn} a_n$$

where k = 1, 2, ..., N

$$B^{kn} = \left(\psi_i^k, U_j \frac{\partial \psi_i^n}{\partial x_j} + \psi_j^n \frac{\partial U_i}{\partial x_j}\right)$$ → Velocity-strain rate interactions

$$C^{knm} = \left(\psi_i^k, \psi_j^n \frac{\partial \psi_i^m}{\partial x_j}\right)$$ → Fluctuating velocity interactions

$$D^{kn} = \left(\psi_i^k, \frac{\partial^2 \psi_i^k}{\partial x_j^2}\right)$$ → Viscous term

The Non-Linear POD Model

- The simplest approach to study observability is to consider its linearization about the desired fixed point $a_k = 0$. The linearized state-space equation is:

$$\frac{da_k}{dt} = J_k \cdot a_n = (-B^{kn} + \frac{1}{Re_D} D^{kn}) \cdot a_n$$

- **Let** the measurement equation be as follows:

$$Y_S = CA_K$$

where $S = 1,2\ldots.M$ is the number of sensors

Y_S is the measurement vector

a_k is the state vector of containing the time-dependent coefficients of the POD model

J_k is the Jacobian of the above nonlinear system

C is called the observation matrix

Conditions for Observability

- Definition: *The pair (J, C) is state observable if and only if the state $a_k(t = 0)$ can be determined from the knowledge of control f_a and the measurement vector* Y_S *over a finite time interval $0 < t < T$.*
- We will demand that the following algebraic condition for observability be met:

$$\mathrm{rank}(C' \vdots J'C' \vdots J'^2 C' \vdots \cdots \vdots J'^{n-1} C') = n^*$$

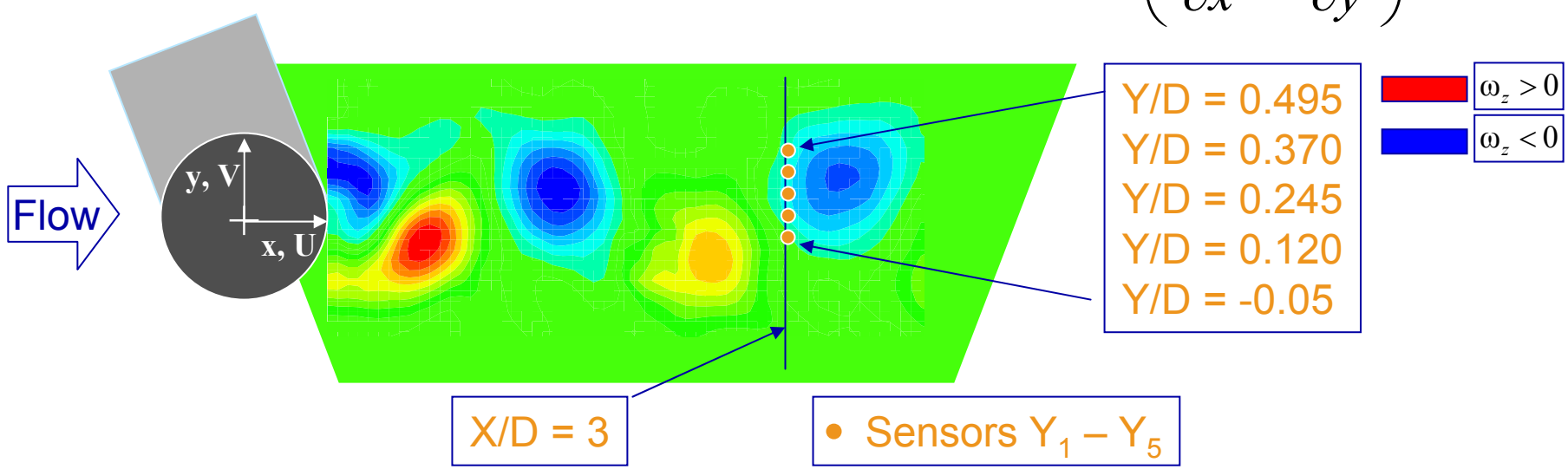
Where C' and J' are the transposes of C and J respectively, n* is the number of modes

Experimental Verification of Observability

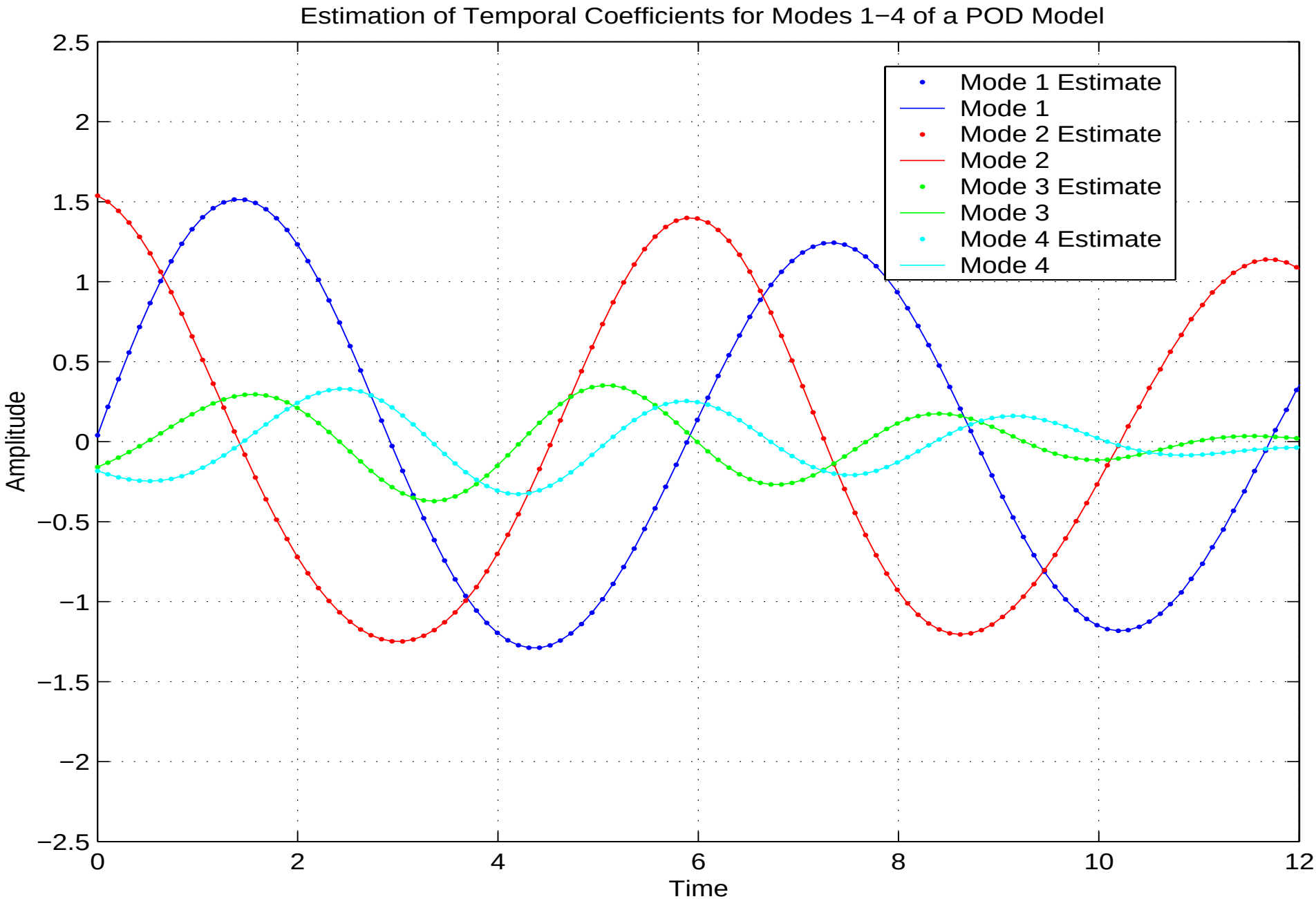
- In order to proceed with the proposed approach, we need to **ensure** the validity of the linearized POD model (matrix J) and the linearity of the measurement equation (matrix C).
- By meeting the conditions for hyperbolicity, we know that the Jacobian, J, captures the vital information concerning the stability of the system (Hartman-Grobman theorem).
- A modified linear stochastic estimation procedure was employed to obtain the mapping of the temporal coefficients of the POD model to the above five sensor readings of the stream-wise velocities.
- In order to validate the accuracy of the Observation Matrix C, we will use it to reproduce the sensor readings, Y_1-Y_5, by multiplying C by the time histories of the 4 modes, a_1-a_4.

PIV Measurements at Re = 125

- Velocity field measurements were made with Particle Image Velocimetry (U, V)
  - Seed water w/ small particles (10μm diameter)
  - Illuminate particles & take two, short-exposure photographs separated in time by a small, known Δt
  - Particle displacements Δx & Δy divided by Δt give velocities (U,V)
  - Data images were acquired at 15 Hz, asynchronously
    - Contour plots show cross-span vorticity, $\omega_z = \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right)$



Mode Estimates based on 5 Sensor Signals



The Observability Test

$$C = \begin{bmatrix} -0.0023 & 0.0387 & -0.1519 & 0.3092 \\ 0.0082 & -0.0056 & -0.1781 & 0.3432 \\ -0.0091 & 0.0810 & -0.1460 & 0.2761 \\ 0.0181 & -0.0511 & -0.1899 & 0.3449 \\ -0.0179 & 0.1195 & -0.1552 & 0.2042 \end{bmatrix} \qquad J = \begin{bmatrix} -0.0318 & 1.1103 & 0.0187 & -0.0005 \\ -1.0220 & -0.0071 & -0.0138 & 0.0072 \\ 0.0010 & -0.0016 & -0.1467 & -1.6666 \\ -0.0042 & -0.0041 & 1.6553 & -0.0962 \end{bmatrix}$$

The numerical values of matrices J and C are used to calculate the rank of the observability matrix.

$$\mathrm{rank}(C' \vdots J'C' \vdots J'^2 C' \vdots J'^3 C') = 4$$

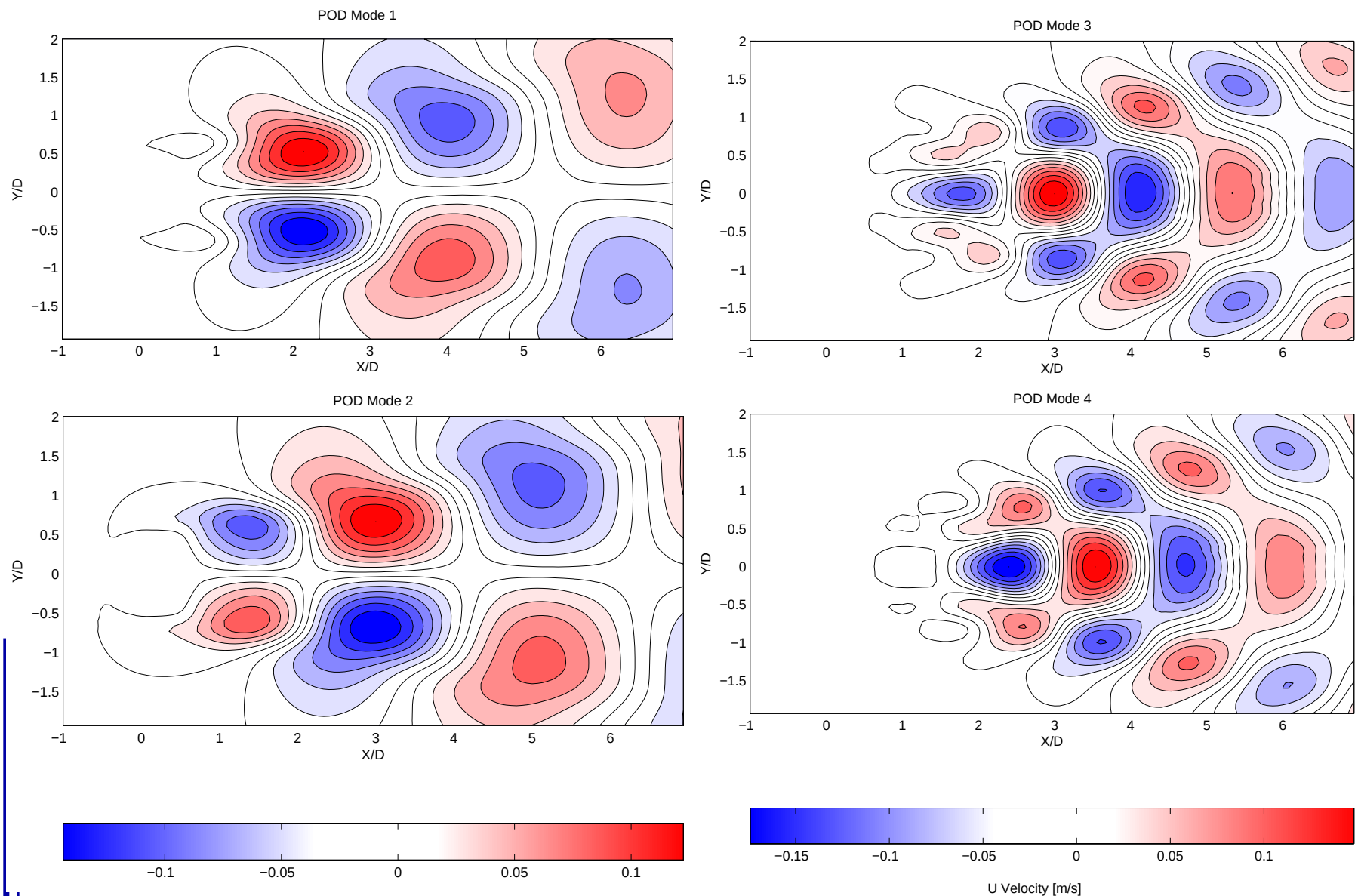
Where C' and J' are the transposes of C and J respectively

Since the rank of the observability matrix is 4, the pair (J,C) is observable and design of an observer is possible.

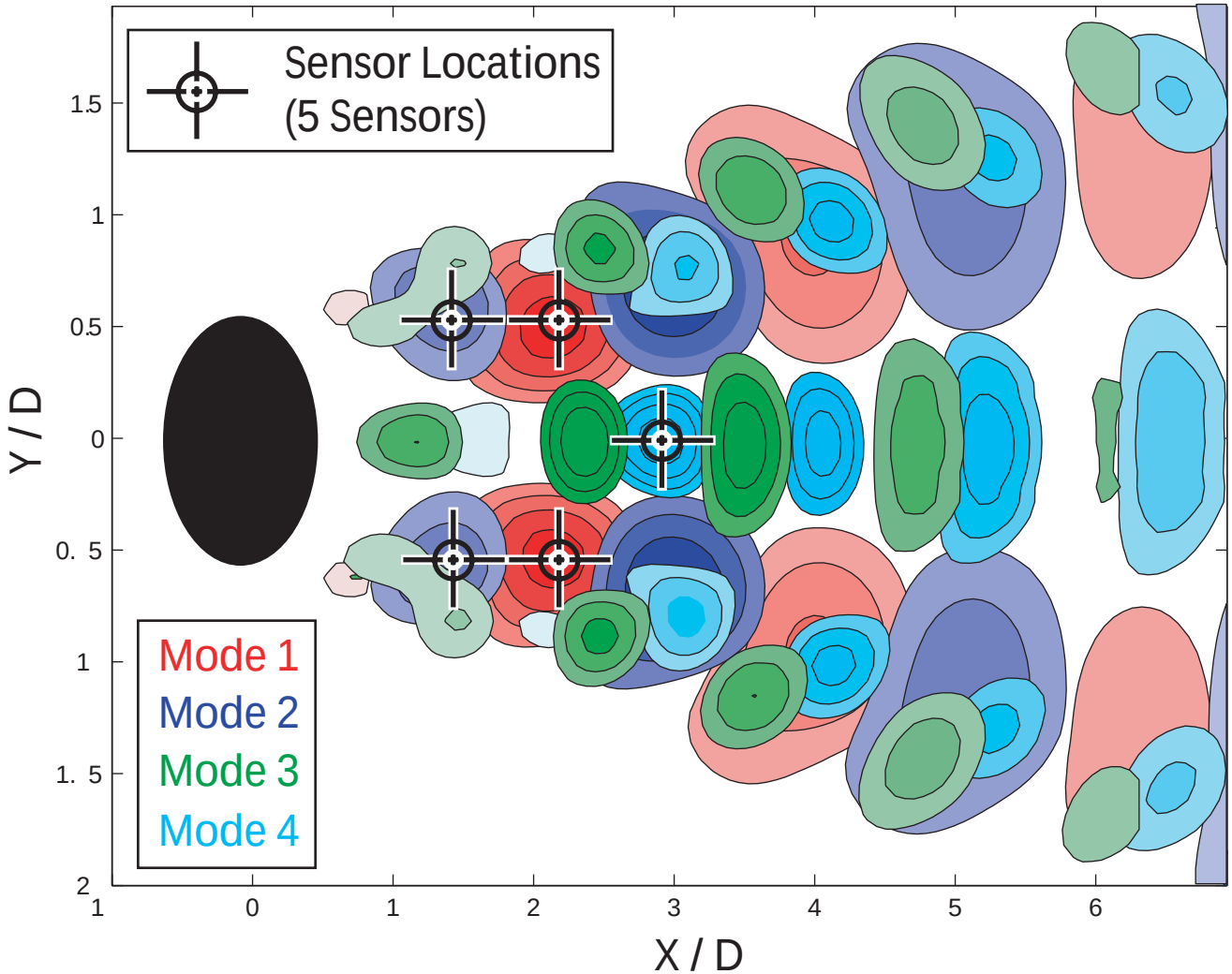
Sensor Placement and Number

- We now examine the sensitivity of the linear stochastic estimator to sensor placement and number.
- The wake of a two dimensional cylinder at Re =100 was obtained from Direct Navier Stokes CFD simulations using COBALT.
- Sensors were placed on the maxima/ minima of the spatial mode distributions.
- Four sensor configurations were studied having one, two, four and five sensors.

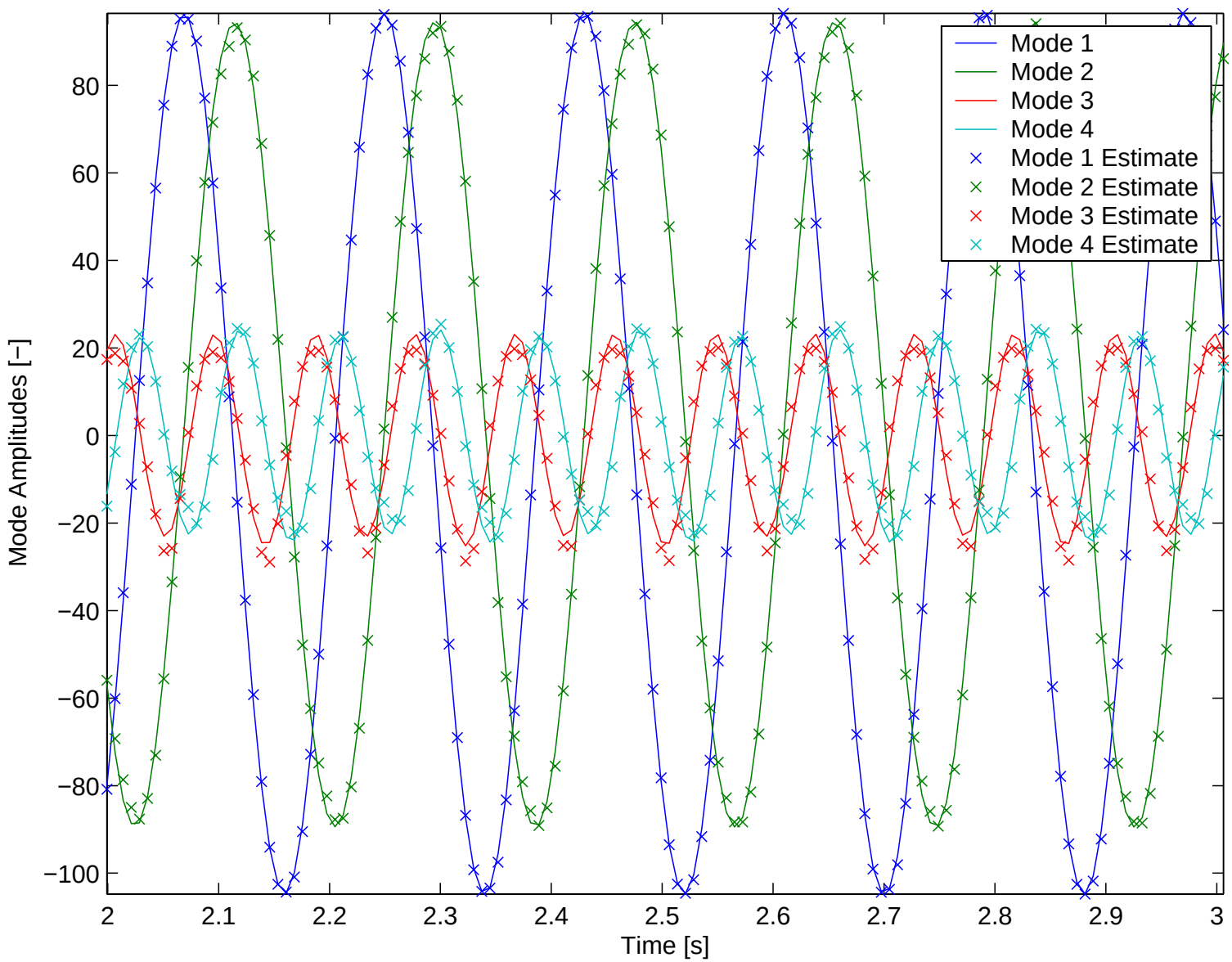
POD Modes CFD



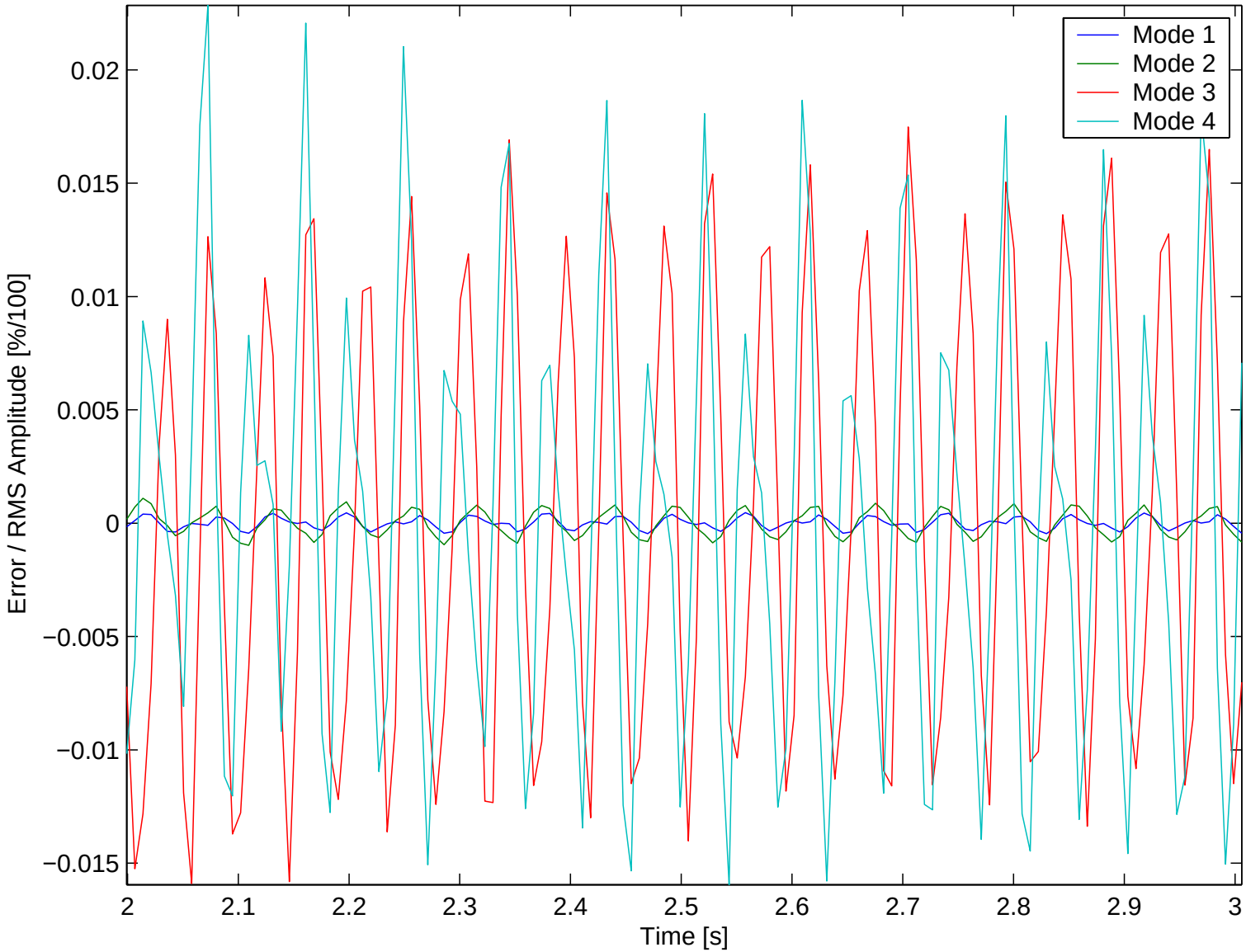
Sensor Location 5 Sensors



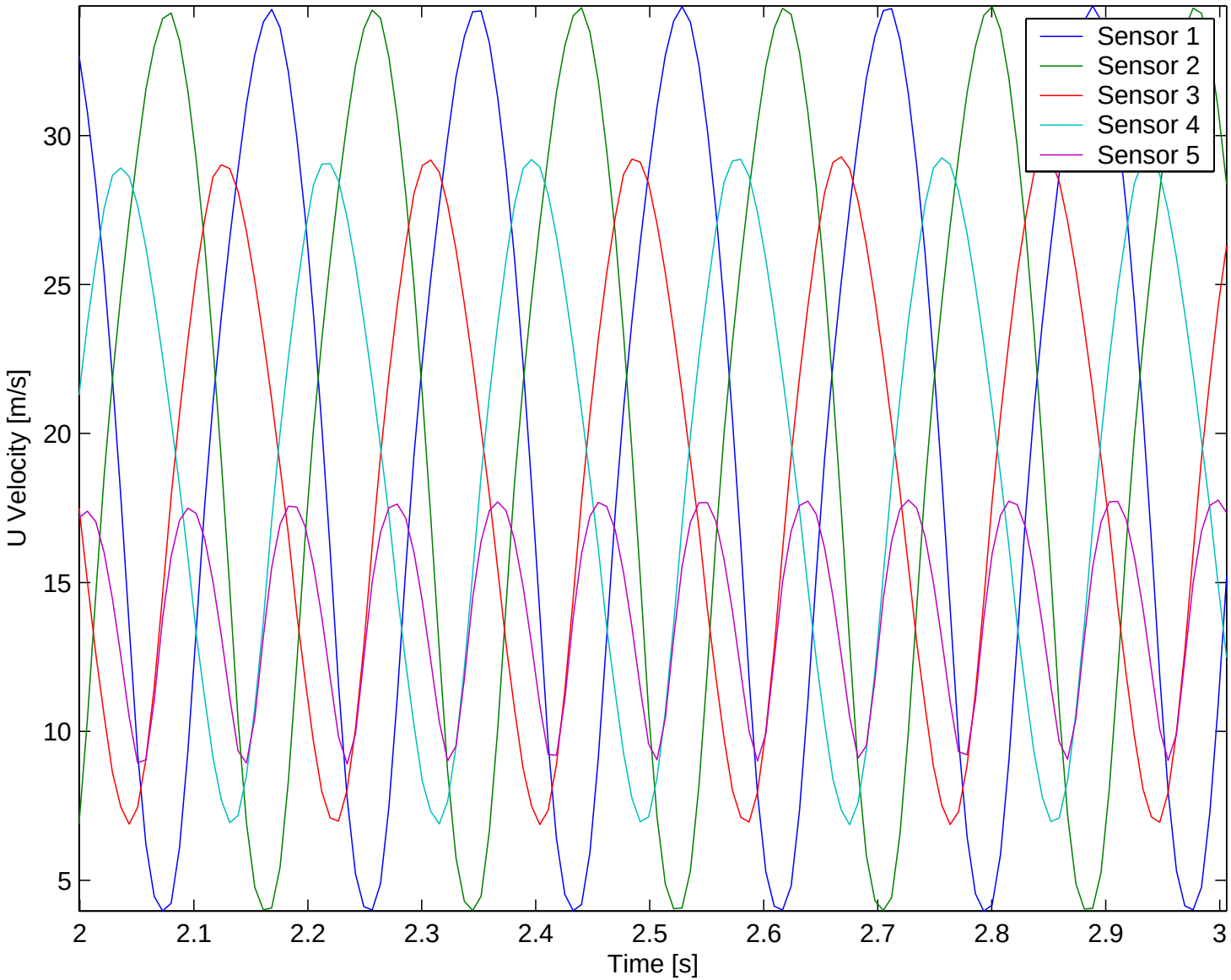
Mode Estimates 5 Sensors



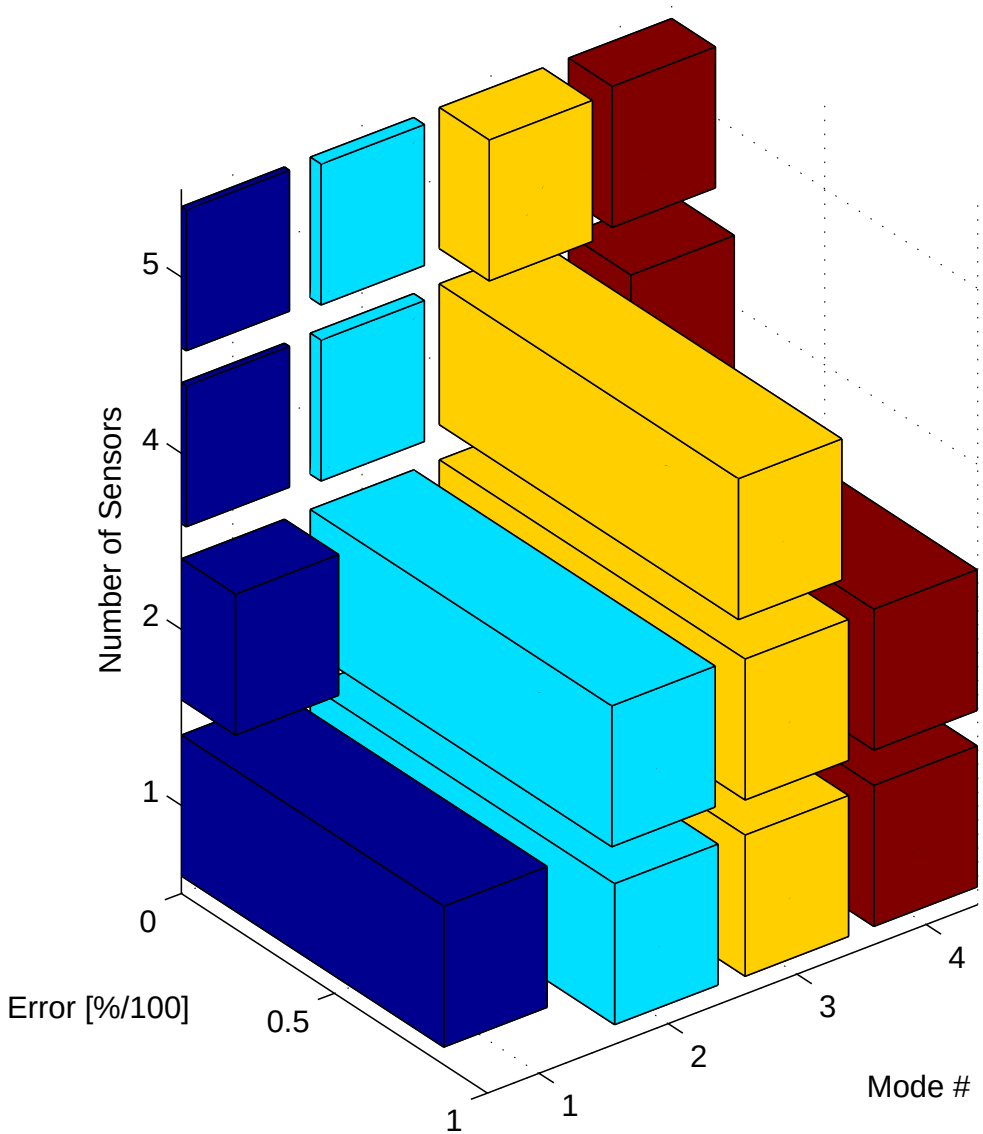
5 Sensors Errors



5 Sensor Time Signals



Error Comparison



Conclusions

- Conditions for observability of a POD model were developed. This test may be implemented providing there is an appropriate linearized state-space model and an accurate linear measurement equation .
- The observation matrix of the linear measurement equation was developed using the linear stochastic estimation procedure.
- Experimental water tunnel data was used to validate the effectiveness of the proposed observability test.
- A computational analysis, based on CFD simulations, was conducted to examine sensitivity to sensor number and placement for the estimation of the temporal coefficients of the low-dimensional POD model.
- Experimental and computational results verify the effectiveness of the developed tools for estimation of the low-dimensional POD model states in real-time.